國 立 雲 林 科 技 大 學

114 學年度第二學期工學院會考試題

時間：10:10~12:00
科目：微積分
班級：

※本次測驗禁止使用電子計算機

1. 求下列積分： $(6\% \times 10 = 60\%)$

(a) $\int \frac{10^{1/x}}{x^2}dx$　(b) $\int \frac{x}{\sqrt{1-4x^4}}dx$　(c) $\int (x^2+2x)e^{2x}\,dx$

(d) $\int x\arcsin x\,dx$　(e) $\int \frac{\sec^4 2x}{\sqrt{\tan 2x}}dx$　(f) $\int \sin 3\theta \sin 5\theta\, d\theta$

(g) $\int \frac{\sqrt{4-x^2}}{x^2}dx$　(h) $\int \frac{1}{x\sqrt{x^2+4}}dx$　(i) $\int \frac{2x^2+x+8}{x^3+4x}dx$

(j) $\int_0^{\infty} \frac{1}{x[1+(\ln x)^2]}dx$

2. 兩函數 $f(x) = \cos x$ 及 $g(x) = 1 - \cos x$ 的曲線在 $-\pi \le x \le \pi$ 定義域範圍內包圍了一個區域，

(a) 求此區域的面積。　(10%)

(b) 求此區域繞直線 $y = 2$ 旋轉所得旋轉體的體積。　(10%)

(c) 求此區域繞直線 $x = 2$ 旋轉所得旋轉體的體積。　(10%)

3. 求以下曲線的弧長。　(10%)

$$y = \frac{x^3}{6} + \frac{1}{2x},\quad 1 \le x \le 3$$

《試題完》

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114 學年度第二學期工學院會考試題

時間：10:10~12:00
科目：Calculus
班級：

※ The use of scientific calculators is not permitted in examinations.

1. Find the integral. (6% × 10 = 60%)

(a) $\int \frac{10^{1/x}}{x^2}dx$ (b) $\int \frac{x}{\sqrt{1-4x^4}}dx$ (c) $\int (x^2+2x)e^{2x}\,dx$

(d) $\int x\arcsin x\,dx$ (e) $\int \frac{\sec^4 2x}{\sqrt{\tan 2x}}dx$ (f) $\int \sin 3\theta \sin 5\theta\, d\theta$

(g) $\int \frac{\sqrt{4-x^2}}{x^2}dx$ (h) $\int \frac{1}{x\sqrt{x^2+4}}dx$ (i) $\int \frac{2x^2+x+8}{x^3+4x}dx$

(j) $\int_0^{\infty} \frac{1}{x[1+(\ln x)^2]}dx$

2. A region is bounded by the graphs of $f(x) = \cos x$ and $g(x) = 1 - \cos x$ in the domain of $-\pi \le x \le \pi$.

(a) Find the area of the region. (10%)

(b) Find the volume of the solid obtained by revolving the region about the line $y = 2$. (10%)

(c) Find the volume of the solid obtained by revolving the region about the line $x = 2$. (10%)

3. Find the exact length of the curve. (10%)

$$y = \frac{x^3}{6} + \frac{1}{2x}, \quad 1 \le x \le 3$$

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標準答案試算表

No. 1

1.

(a) $\int \frac{10^{1/x}}{x^2}dx = -\int 10^{1/x}(-x^{-2})dx = -\frac{10^{1/x}}{\ln 10} + C$

(b) $\int \frac{x}{\sqrt{1-4x^4}}dx = \frac{1}{4}\int \frac{1}{\sqrt{1-(2x^2)^2}}4xdx = \frac{1}{4}\arcsin(2x^2) + C$

(c) Let $dv = e^{2x}dx$ and $u = x^2 + 2x \rightarrow v = \frac{1}{2}e^{2x}$ and $du = (2x+2)\,dx$

$$\int (x^2+2x)e^{2x}\,dx = \frac{1}{2}(x^2+2x)e^{2x} - \int e^{2x}(x+1)\,dx \qquad (1)$$

Let $dv = e^{2x}dx$ and $u = x+1 \rightarrow v = \frac{1}{2}e^{2x}$ and $du = dx$

$$\int e^{2x}(x+1)\,dx = \frac{1}{2}(x+1)e^{2x} - \int \frac{1}{2}e^{2x}dx = \frac{1}{2}(x+1)e^{2x} - \frac{1}{4}e^{2x} \qquad (2)$$

Substitute equation (2) into equation (1).

$$\int (x^2+2x)e^{2x}\,dx = \frac{1}{2}(x^2+2x)e^{2x} - \frac{1}{2}(x+1)e^{2x} + \frac{1}{4}e^{2x}$$

$$= \frac{1}{4}(2x^2+2x-1)e^{2x} + C$$

(d) Let $dv = xdx$ and $u = \arcsin x \rightarrow v = \frac{x^2}{2}$ and $du = \frac{1}{\sqrt{1-x^2}}dx$

$$\int x\arcsin x\,dx = \frac{x^2}{2}\arcsin x - \int \frac{x^2}{2\sqrt{1-x^2}}dx$$

Let $x = \sin\theta$. Then, $dx = \cos\theta\,d\theta$ and $\sqrt{1-x^2} = \cos\theta$

$$\int \frac{x^2}{2\sqrt{1-x^2}}dx = \frac{1}{2}\int \frac{\sin^2\theta}{\cos\theta}\cos\theta\,d\theta = \frac{1}{2}\int \frac{1-\cos 2\theta}{2}d\theta = \frac{1}{4}\left(\theta - \frac{\sin 2\theta}{2}\right)$$

$$= \frac{1}{4}(\theta - \sin\theta\cos\theta) = \frac{1}{4}\left(\arcsin x - x\sqrt{1-x^2}\right)$$

$$\int x\arcsin x\,dx = \frac{x^2}{2}\arcsin x - \frac{1}{4}\arcsin x + \frac{x\sqrt{1-x^2}}{4} + C$$

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標準答案試算表

No. 2

(e) Let $u=\tan 2x$, $du=2\sec^2 2x\,dx$

$$\int \frac{\sec^4 2x}{\sqrt{\tan 2x}}dx = \frac{1}{2}\int \frac{(1+\tan^2 2x)}{\sqrt{\tan 2x}}2\sec^2 2x dx = \frac{1}{2}\int \frac{(1+u^2)}{u^{1/2}}du$$

$$= \frac{1}{2}\int (u^{-1/2}+u^{3/2})du = \frac{1}{2}\left(2u^{1/2}+\frac{2}{5}u^{5/2}\right)+C = \sqrt{\tan 2x}+\frac{1}{5}(\tan 2x)^{5/2}+C$$

(f) $$\int \sin 3\theta \sin 5\theta\, d\theta = \int \frac{1}{2}(\cos 2\theta - \cos 8\theta)d\theta = \frac{1}{4}\sin 2\theta - \frac{1}{16}\sin 8\theta + C$$

(g) Let $x=2\sin\theta$. Then, $dx=2\cos\theta\,d\theta$ and $\sqrt{4-x^2}=2\cos\theta$

$$\int \frac{\sqrt{4-x^2}}{x^2}dx = \int \frac{2\cos\theta}{4\sin^2\theta}2\cos\theta\,d\theta = \int \frac{\cos^2\theta}{\sin^2\theta}d\theta = \int \frac{1-\sin^2\theta}{\sin^2\theta}d\theta$$

$$= \int (\csc^2\theta - 1)\,d\theta = -\cot\theta - \theta = -\frac{\sqrt{4-x^2}}{x} - \arcsin\frac{x}{2} + C$$

(h) Let $x=2\tan\theta$. Then, $dx=2\sec^2\theta\,d\theta$ and $\sqrt{x^2+4}=2\sec\theta$

$$\int \frac{1}{x\sqrt{x^2+4}}dx = \int \frac{1}{4\tan\theta\sec\theta}2\sec^2\theta\,d\theta = \int \frac{\sec\theta}{2\tan\theta}d\theta = \int \frac{1}{2\sin\theta}d\theta$$

$$= \frac{1}{2}\int \csc\theta\,d\theta = -\frac{1}{2}\ln|\csc\theta+\cot\theta| = -\frac{1}{2}\ln\left|\frac{\sqrt{x^2+4}}{x}+\frac{2}{x}\right| + C$$

(i) $$\frac{2x^2+x+8}{x^3+4x} = \frac{2x^2+x+8}{x(x^2+4)} = \frac{A}{x}+\frac{Bx+C}{x^2+4} = \frac{2}{x}+\frac{1}{x^2+4}$$

$$\int \frac{2x^2+x+8}{x^3+4x}dx = \int\left(\frac{2}{x}+\frac{1}{x^2+4}\right)dx = 2\ln|x| + \frac{1}{2}\arctan\frac{x}{2} + C$$

(j) $$\int_0^\infty \frac{1}{x[1+(\ln x)^2]}dx = \lim_{a\to 0^+}\int_a^1 \frac{1}{1+(\ln x)^2}\cdot\frac{1}{x}dx + \lim_{b\to\infty}\int_1^b \frac{1}{1+(\ln x)^2}\cdot\frac{1}{x}dx$$

$$\int_0^\infty \frac{1}{1+(\ln x)^2}\cdot\frac{1}{x}dx = \lim_{a\to 0^+}\left[\arctan(\ln x)\right]_a^1 + \lim_{b\to\infty}\left[\arctan(\ln x)\right]_1^b = 0-\left(-\frac{\pi}{2}\right)+\frac{\pi}{2}-0$$

$$= \pi$$

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標準答案試算表

No. 3

2. 兩函數 $f(x)=\cos x$ 及 $g(x)=1-\cos x$ 的曲線在 $-\pi\le x\le\pi$ 定義域範圍內包圍了一個區域，

(a) 求此區域的面積。 (10%)

(b) 求此區域繞直線 $y=2$ 旋轉所得旋轉體的體積。 (10%)

(c) 求此區域繞直線 $x=2$ 旋轉所得旋轉體的體積。 (10%)

The point of intersection: $\cos x=1-\cos x\Rightarrow\cos x=\dfrac{1}{2}\Rightarrow x=\pm\dfrac{\pi}{3}$

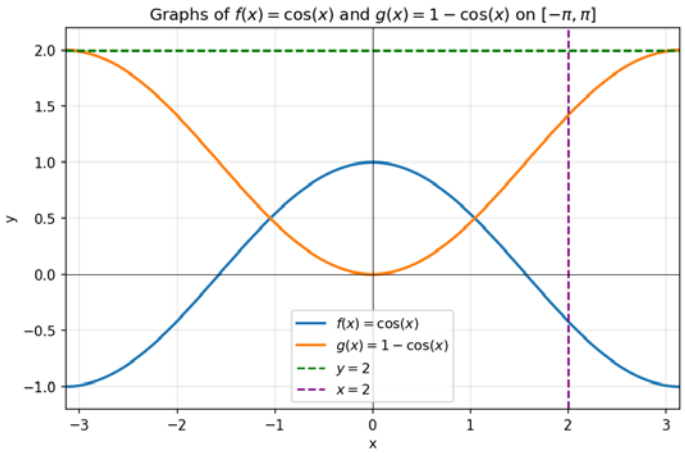
(a) Area

The distance between curves: $\cos x-(1-\cos x)=2\cos x-1$

$$A=\int_{-\pi/3}^{\pi/3}(2\cos x-1)\,dx=\left[\,2\sin x-x\right]_{-\pi/3}^{\pi/3}$$

$$=\left(2\cdot\frac{\sqrt{3}}{2}-\frac{\pi}{3}\right)-\left[2\cdot\left(-\frac{\sqrt{3}}{2}\right)-\left(-\frac{\pi}{3}\right)\right]$$

$$=2\sqrt{3}-\frac{2\pi}{3}$$



(b) Use the disk method

$$r_{outer}=2-(1-\cos x)=1+\cos x$$

$$r_{inner}=2-\cos x$$

$$V=\pi\int_{-\pi/3}^{\pi/3}\left[(1+\cos x)^2-(2-\cos x)^2\right]dx$$

$$=\pi\int_{-\pi/3}^{\pi/3}\left[1+2\cos x+\cos^2 x-(4-4\cos x+\cos^2 x)\right]dx$$

$$=\pi\int_{-\pi/3}^{\pi/3}(-3+6\cos x)\,dx=\pi\left[-3x+6\sin x\right]_{-\pi/3}^{\pi/3}$$

$$=\pi\left[-\pi+3\sqrt{3}-(\pi-3\sqrt{3})\right]=-2\pi^2+6\sqrt{3}\pi$$

(c) Use the cylindrical shell method

$$V=\int_{-\pi/3}^{\pi/3}2\pi(2-x)\left[\cos x-(1-\cos x)\right]dx=2\pi\int_{-\pi/3}^{\pi/3}(2-x)(2\cos x-1)\,dx$$

$$=2\pi\int_{-\pi/3}^{\pi/3}(-2+x+4\cos x-2x\cos x)\,dx$$

$$=2\pi\left[-2x+\frac{x^2}{2}+4\sin x-2x\sin x-2\cos x\right]_{-\pi/3}^{\pi/3}$$

$$=2\pi\left[-\frac{4\pi}{3}+4\sqrt{3}\right]=-\frac{8}{3}\pi^2+8\sqrt{3}\pi$$

3. 求以下曲線的弧長。 (10%)

$$y=\frac{x^3}{6}+\frac{1}{2x},\quad 1\le x\le 3$$

$$y'=\frac{3x^2}{6}-\frac{1}{2x^2}=\frac{1}{2}\left(x^2-\frac{1}{x^2}\right)$$

$$1+(y')^2=1+\frac{1}{4}\left(x^4-\frac{1}{2}+\frac{1}{x^4}\right)=\frac{1}{4}\left(x^4+\frac{1}{2}+\frac{1}{x^4}\right)=\left[\frac{1}{2}\left(x^2+\frac{1}{x^2}\right)\right]^2$$

$$s=\int_1^3\sqrt{1+(y')^2}\,dx=\int_1^3\frac{1}{2}\left(x^2+\frac{1}{x^2}\right)dx$$

$$=\frac{1}{2}\left[\frac{x^3}{3}-\frac{1}{x}\right]_1^3=\frac{1}{2}\left[9-\frac{1}{3}-\left(\frac{1}{3}-1\right)\right]$$

$$=\frac{1}{2}\left(10-\frac{2}{3}\right)=\frac{14}{3}$$