國 立 雲 林 科 技 大 學

114 學年度第一學期工學院會考試題

時間：2026/01/07 10:10~12:00
科目：微積分(一)
班級：

❖ 本次會考**禁止使用計算機。**

1. 求$f(x)$在指定區間的絕對極大值(absolute maximum)與絕對極小值(absolute minimum)。

$(a)\ f(x)=x^{\frac{2}{3}}\left(\frac{5}{2}-x\right),\quad [-2,2]$ (10%)

$(b)\ f(x)=x-2\sin x,\quad [0,2\pi]$ (10%)

2. 函數 $f(x)=\frac{8x}{x^2+4}$，求 (20%)

(a) 遞增(increasing)與遞減(decreasing)的開放區間以及相對高點與相對低點的座標位置。

(b) 凹口向下(concave down)與凹口向上(concave up)的開放區間以及反曲點的座標位置。

3. 試求下列所示之極限值。

$(a)\lim_{x\to 0}\left[\ln(1-\cos x)-\ln\left(\frac{x^2}{2}\right)\right]$ (5%)

$(b)\lim_{x\to 0^+}\left[\frac{1}{\ln(1-\sin x)}-\frac{1}{\sin x}\right]$ (5%)

4. 試求下列函數之不定積分。

$(a)\int\sqrt{x}(1+x^2)^2\,dx$ (5%) $(b)\int\frac{\sin 2x}{\sin^2 x}dx$ (5%)

$(c)\int\frac{(a+bx^2)}{\sqrt{3ax+bx^3}}dx$ (5%) $(d)\int e^{\tan 2x}\sec^2 2x\,dx$ (5%)

$(e)\int\sqrt{\cot x}\csc^2 x\,dx$ (5%)

5. 試計算下列函數之定積分。

$(a)\int_0^1(1-x^2)^3\,dx$ (5%) $(b)\int_0^{\frac{3\pi}{2}}|\sin x|\,dx$ (5%)

$(c)\int_{\frac{1}{2}}^1\frac{\cos(1/x^2)}{x^3}dx$ (5%) $(d)\int_1^4\frac{(\ln x)^3}{x}dx$ (5%)

$(e)\frac{d}{dx}\int_{\sqrt{x}}^x\frac{\cos t}{t}dt$ (5%)

❖ The use of scientific calculators is not permitted in examinations.

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時間：2026/01/07 10:10~12:00

科目：Calculus (I)

班級：

1. Find the absolute maximum and the absolute minimum values on the given interval.

$(a)\ f(x) = x^{\frac{2}{3}}\left(\frac{5}{2} - x\right), \quad [-2, 2]$ (10%)

$(b)\ f(x) = x - 2\sin x, \quad [0, 2\pi]$ (10%)

2. Given a function $f(x) = \frac{8x}{x^2+4}$

(a) Find the increasing intervals, the decreasing intervals, relative maximum values and relative minimum values. (10%)

(b) Find the interval of concavity and inflection points. (10%)

3. Evaluate

$(a)\ \lim_{x\to 0}\left[\ln(1-\cos x) - \ln\frac{x^2}{2}\right]$ (5%)

$(b)\ \lim_{x\to 0^+}\left[\frac{1}{\ln(1-\sin x)} - \frac{1}{\sin x}\right]$ (5%)

4. Evaluate each of the following indefinite integrals.

$(a)\int \sqrt{x}(1+x^2)^2\,dx$ (5%) $(b)\int \frac{\sin 2x}{\sin^2 x}dx$ (5%)

$(c)\int \frac{(a+bx^2)}{\sqrt{3ax+bx^3}}dx$ (5%) $(d)\int e^{\tan 2x}\sec^2 2x\,dx$ (5%)

$(e)\int \sqrt{\cot x}\csc^2 x\,dx$ (5%)

5. Evaluate each of the following definite integrals.

$(a)\int_0^1 (1-x^2)^3\,dx$ (5%) $(b)\int_0^{\frac{3\pi}{2}} |\sin x|\,dx$ (5%)

$(c)\int_{\frac{1}{2}}^1 \frac{\cos(1/x^2)}{x^3}dx$ (5%) $(d)\int_1^4 \frac{(\ln x)^3}{x}dx$ (5%)

$(e)\frac{d}{dx}\int_{\sqrt{x}}^x \frac{\cos t}{t}dt$ (5%)

1. Find the absolute maximum and the absolute minimum values on the given interval.

(a) 10 分

$$f(x) = x^{\frac{2}{3}}\left(\frac{5}{2} - x\right), \quad [-2, 2]$$

Sol.

$$f'(x) = \frac{d}{dx}\left[x^{\frac{2}{3}}\right]\left(\frac{5}{2} - x\right) + x^{\frac{2}{3}} \cdot \frac{d}{dx}\left[\frac{5}{2} - x\right] = \frac{2}{3}x^{-\frac{1}{3}}\left(\frac{5}{2} - x\right) + x^{\frac{2}{3}}(0 - 1)$$
$$= x^{-\frac{1}{3}}\left[\frac{5}{3} - \frac{2}{3}x - x\right] = x^{-\frac{1}{3}}\left(\frac{5}{3} - \frac{5}{3}x\right)$$

Set $f'(x) = x^{-\frac{1}{3}}\left(\frac{5}{3} - \frac{5}{3}x\right) = 0$, the critical numbers are $x = 0, x = 1$

$x = -2, f(-2) = (-2)^{\frac{2}{3}}\left(\frac{5}{2} - (-2)\right) = \sqrt[3]{4} \cdot \frac{9}{2} = \frac{9\sqrt[3]{4}}{2}$

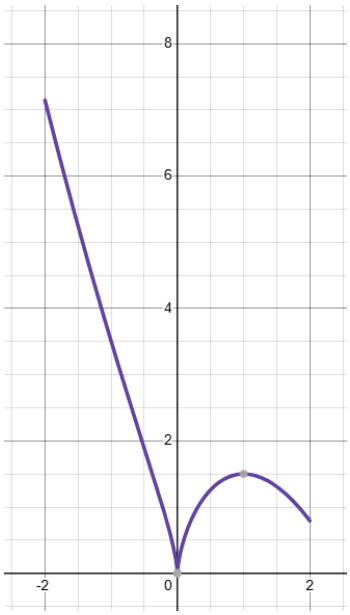
$x = 0, f(0) = 0^{\frac{2}{3}} \cdot \left(\frac{5}{2} - 0\right) = 0$

$x = 1, f(1) = 1^{\frac{2}{3}} \cdot \left(\frac{5}{2} - 1\right) = \frac{3}{2}$

$x = 2, f(2) = 2^{\frac{2}{3}}\left(\frac{5}{2} - 2\right) = \sqrt[3]{4} \cdot \frac{1}{2} = \frac{\sqrt[3]{4}}{2}$

The absolute maximum is $x = -2$: $(-2, \frac{9\sqrt[3]{4}}{2})$

The absolute minimum is $x = 0$: $(0, 0)$



1. Find the absolute maximum and the absolute minimum values on the given interval.

(b) 10 分

$$f(x) = x - 2\sin(x), \quad [0, 2\pi]$$

Sol:

$$f'(x) = \frac{d}{dx}[x] - 2\cos(x) = \frac{1}{2} - 2\cos(x)$$

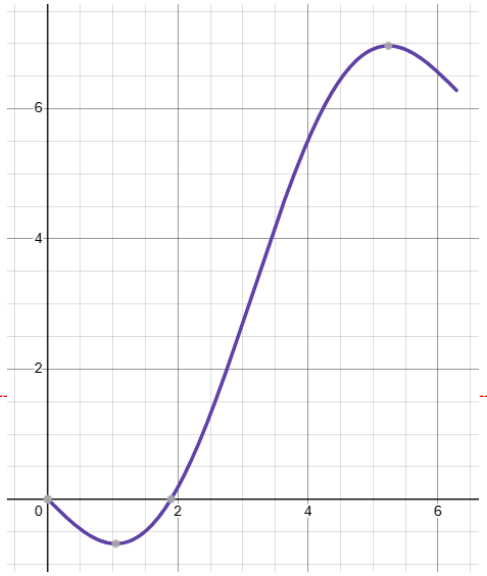
Set $f'(x) = 1 - 2\cos(x) = 0$, then $\cos(x) = \frac{1}{2}, x = \frac{\pi}{3}, x = \frac{5}{3}\pi$

$x = 0, f(0) = 0 - 2\sin(0) = 0 - 2 \cdot 0 = 0$

$x = \frac{\pi}{3}, f\left(\frac{\pi}{3}\right) = \frac{\pi}{3} - 2\sin\left(\frac{\pi}{3}\right) = \frac{\pi}{3} - 2 \cdot \frac{\sqrt{3}}{2} = \frac{\pi}{3} - \sqrt{3}$

$x = \frac{5}{3}\pi, f\left(\frac{5}{3}\pi\right) = \frac{5}{3}\pi - 2\sin\left(\frac{5}{3}\pi\right) = \frac{5}{3}\pi + \sqrt{3}$

$x = 2\pi, f(2\pi) = 2\pi + 2\sin(2\pi) = 2\pi + 0 = 2\pi$



The absolute maximum is $x=\frac{5}{3}\pi:(\frac{5}{3}\pi,\frac{5}{3}\pi+\sqrt{3})$

The absolute minimum is $x=\frac{\pi}{3}:(\frac{\pi}{3},\frac{\pi}{3}-\sqrt{3})$

2. (a) Find the increasing intervals, the decreasing intervals, relative maximum values and relative minimum values (10 分)

$$f(x)=\frac{8x}{x^2+4}$$

Sol.

$$f'(x)=\frac{8(x^2+4)-8x(2x+0)}{(x^2+4)^2}=\frac{8x^2+32-16x^2}{(x^2+4)^2}=\frac{-8\cdot(x^2-4)}{(x^2+4)^2}$$

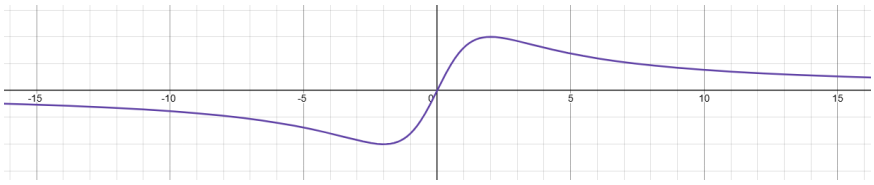
Set $f'(x)=\frac{-8(x^2-4)}{(x^2+4)^2}=0$, the critical numbers are $x=2,x=-2$

The increasing interval is $(-2,2)$

The decreasing intervals are $(-\infty,-2)\cup(2,\infty)$

The relative maximum is $x=2,y=\frac{8\cdot2}{2^2+4}=\frac{16}{16}=1$

The relative minimum is $x=-2,y=\frac{8\cdot(-2)}{(-2)^2+4}=-\frac{16}{16}=-1$



2. (b) Find the interval of concavity and inflection points (10 分)

$$f(x)=\frac{8x}{x^2+4}$$

Sol.

$$\begin{aligned}f''(x)&=\frac{-8\cdot2x\cdot(x^2+4)^2-(-8)(x^2-4)\cdot2(x^2+4)(2x+0)}{[(x^2+4)^2]^2}\\&=\frac{-16x(x^2+4)^2+32x(x^2-4)(x^2+4)}{(x^2+4)^4}\\&=\frac{(x^2+4)[-16x(x^2+4)+32x(x^2-4)]}{(x^2+4)^4}\\&=\frac{(x^2+4)[-16x^3-64x+32x^3-128x]}{(x^2+4)^4}=\frac{(x^2+4)[16x^3-192x]}{(x^2+4)^4}\\&=\frac{16x(x+4)(x^2-12)}{(x^2+4)^4}=\frac{16x(x^2-12)}{(x^2+4)^3}\end{aligned}$$

Set $f''(x)=\frac{16x(x^2-12)}{(x^2+4)^3}=0$, the possible inflection points are $x=0, x=+2\sqrt{3}, x=-2\sqrt{3}$

The concave upward intervals are $(-2\sqrt{3},0)\cup(2\sqrt{3},\infty)$

The concave downward intervals are $(-\infty,-2\sqrt{3})\cup(0,2\sqrt{3})$

The inflection point are

$x=0, y=\frac{8\cdot 0}{0^2+4}=\frac{0}{4}=0$

$x=2\sqrt{3}, y=\frac{8\cdot 2\sqrt{3}}{(2\sqrt{3})^2+4}=\frac{16\sqrt{3}}{12+4}=\sqrt{3}$

$x=-2\sqrt{3}, y=\frac{8\cdot(-2\sqrt{3})}{(-2\sqrt{3})^2+4}=\frac{-16\sqrt{3}}{12+4}=-\sqrt{3}$

3. Evaluate

(a) 5 分

$$\lim_{x\to 0}[\ln(1-\cos x)-\ln\left(\frac{x^2}{2}\right)]=\ln(1-1)-\ln\left(\frac{0^2}{2}\right)=\ln\ 0-\ln\ 0=(-\infty)-(-\infty)$$

$$=\lim_{x\to 0}\ln\frac{1-\cos x}{\frac{1}{2}x^2}=\ln\left(\lim_{x\to 0}\frac{1-1}{\frac{1}{2}\cdot 0^2}\right)=\ln\left(\frac{0}{0}\right)$$

$$\xrightarrow{L'Hopital}\ln\left(\lim_{x\to 0}\frac{\frac{d}{dx}[1-\cos x]}{\frac{d}{dx}\left[\frac{1}{2}x^2\right]}\right)=\ln\left(\lim_{x\to 0}\frac{0-(-\sin x)}{\frac{1}{2}\cdot 2x}\right)=\ln\left(\lim_{x\to 0}\frac{\sin x}{x}\right)=\ln\left(\frac{0}{0}\right)$$

$$\xrightarrow{L'Hopital}\ln\left(\lim_{x\to 0}\frac{\frac{d}{dx}[\sin x]}{\frac{d}{dx}[x]}\right)=\ln\left(\lim_{x\to 0}\frac{\cos x}{1}\right)=\ln\left(\frac{1}{1}\right)=\ln\ 1=0$$

3. Evaluate

(b) 5 分

$$\lim_{x\to 0^+}\left[\frac{1}{\ln(1-\sin x)}-\frac{1}{\sin x}\right]=\frac{1}{\ln(1-0)}-\frac{1}{0}=\infty-\infty$$

$$=\lim_{x\to 0^+}\frac{\sin x-\ln(1-\sin x)}{\ln(1-\sin x)\sin x}=\frac{0}{0}\xrightarrow{L'Hopital}$$

$$=\lim_{x\to 0^+}\frac{\frac{d}{dx}[\sin x-\ln(1-\sin x)]}{\frac{d}{dx}[\ln(1-\sin x)\sin x]}$$

$$=\lim_{x\to 0^+}\frac{\cos x-\frac{1}{1-\sin x}\cdot[0-\cos x]}{\frac{1}{1-\sin x}\cdot[0-\cos(x)]\cdot\sin x+\ln(1-\sin x)\cdot\cos x}$$

$$=\frac{0-\frac{1}{1-0}\cdot[0-1]}{\frac{1}{1-0}\cdot[0-1]\cdot 0+\ln(1-0)\cdot 1}=\frac{0-(-1)}{0+0}=\frac{1}{0}=\infty$$

4. Evaluate each of the following indefinite integrals.

(a) 5 分

$$\int\sqrt{x}(1+x^2)^2dx=\int x^{\frac{1}{2}}\cdot(1+2x^2+x^4)\,dx$$

$$=\int x^{\frac{1}{2}}+2x^{\frac{5}{2}}+x^{\frac{9}{2}}dx$$

$$=\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}+2\cdot\frac{x^{\frac{5}{2}+1}}{\frac{5}{2}+1}+\frac{x^{\frac{9}{2}+1}}{\frac{9}{2}+1}$$

$$=\mathbf{\frac{2}{3}x^{\frac{3}{2}}+\frac{4}{7}x^{\frac{7}{2}}+\frac{2}{11}x^{\frac{11}{2}}+c}$$

4. Evaluate each of the following indefinite integrals.

(b) 5 分

$$\int\frac{\sin 2x}{\sin^2 x}dx=\int\frac{2\sin x\cos x}{(\sin x)^2}dx=2\int\frac{\cos x}{\sin x}dx=2\int\cot x\,dx=2\cdot\ln|\sin x|+c$$

4. Evaluate each of the following indefinite integrals.

(c) 5 分

$$\int \frac{(a+bx^2)}{\sqrt{3ax+bx^3}}dx$$

Let $u = 3ax + bx^3, du = (3a + 3bx^2)dx, dx = \frac{du}{3a+3bx^2}$

$$\int \frac{(a+bx^2)}{\sqrt{ax+bx^3}}dx = \int \frac{(a+bx^2)}{\sqrt{u}} \cdot \frac{du}{3a+3bx^2} = \int \frac{(a+bx^2)}{3(a+bx^2)} \cdot \frac{1}{u^{\frac{1}{2}}}du$$

$$= \frac{1}{3}\int u^{-\frac{1}{2}}du = \frac{1}{3} \cdot \frac{u^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} = \frac{1}{3} \cdot 2u^{\frac{1}{2}} = \frac{2}{3}\sqrt{3ax+bx^3} + c$$

4. Evaluate each of the following indefinite integrals.

(d) 5 分

$$\int e^{\tan 2x} \sec^2 2x\, dx$$

Let $u = \tan 2x\,, du = \sec^2(2x) \cdot \frac{d}{dx}[2x] = 2\sec^2(2x)\,dx, dx = \frac{du}{2\sec^2(2x)}$

$$\int e^{\tan 2x} \sec^2 2x\, dx = \int e^u \cdot \sec^2 2x \cdot \frac{du}{2\sec^2 2x} = \frac{1}{2}\int e^u\, du = \frac{1}{2}e^u = \frac{1}{2}e^{\tan(2x)} + c$$

4. Evaluate each of the following indefinite integrals.

(e) 5 分

$$\int \sqrt{\cot x}\csc^2 x\,dx$$

Let $u=\cot x, du=-\csc^2 x\,dx, dx=\frac{-du}{\csc^2 x}$

$$\int \sqrt{\cot x}\cdot\csc^2 x\,dx=\int \sqrt{u}\cdot\csc^2 x\cdot\frac{-du}{\csc^2 x}=-\int u^{\frac{1}{2}}du=-\frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1}=-\frac{2}{3}u^{\frac{3}{2}}$$

$$=-\frac{2}{3}\sqrt{\cot^3 x}+c$$

5. Evaluate each of the following definite integrals.

(a) 5 分

$$\int_0^1 (1-x^2)^3\,dx=\int_0^1 (1-3x^2+3x^4-x^6)\,dx$$

$$=\left[x-3\cdot\frac{x^{2+1}}{2+1}+3\cdot\frac{x^{4+1}}{4+1}-\frac{x^{6+1}}{6+1}\right]_0^1=\left[x-x^3+\frac{3}{5}x^5-\frac{1}{7}x^7\right]_0^1$$

$$=\left[\left(1-1^3+\frac{3}{5}\cdot 1^5-\frac{1}{7}\cdot 1^7\right)-\left(0-0^3+\frac{3}{5}\cdot 0^5-\frac{1}{7}\cdot 0^7\right)\right]$$

$$=\left[\left(1-1+\frac{3}{5}-\frac{1}{7}\right)-0\right]=\frac{21-5}{35}=\frac{16}{35}$$

5. Evaluate each of the following definite integrals.

(b) 5 分

$$\int_0^{\frac{3}{2}\pi}|\sin x|dx=\int_0^{\pi}\sin x\,dx+\int_{\pi}^{\frac{3}{2}\pi}(-\sin x)\,dx=[-\cos x]_0^{\pi}-[-\cos x]_{\pi}^{\frac{3}{2}\pi}$$

$$=[-\cos(\pi)-[-\cos(0)]]-\left[-\cos\left(\frac{3}{2}\pi\right)-[-\cos(\pi)]\right]$$

$$=[-(-1)+1]-[0+(-1)]=1+1-0+1=3$$

5. Evaluate each of the following definite integrals.

(c) 5 分

$$\int_{\frac{1}{2}}^{1} \frac{\cos\,(1/x^2)}{x^3}\,dx$$

Let $u = \frac{1}{x^2} = x^{-2}, du = -2x^{-3}dx, dx = -\frac{1}{2}x^3 du$

$$\int_{\frac{1}{2}}^{1} \frac{\cos(1/x^1)}{x^3}\,dx = \int_{u=4}^{u=1} \frac{\cos\,(u)}{x^3}\cdot\left(-\frac{1}{2}x^3 du\right) = \frac{1}{2}\int_{u=1}^{u=4} \cos(u)\,du$$

$$= \frac{1}{2}\cdot[\sin(u)]_{u=1}^{u=4} = \frac{1}{2}[\sin(4) - \sin(1)]$$

5. Evaluate each of the following definite integrals.

(d) 5 分

$$\int_{1}^{4} \frac{(\ln x)^3}{x}\,dx$$

Let $u = \ln x\,, du = \frac{1}{x}dx, dx = xdu$

$$\int_{1}^{4} \frac{(\ln x)^3}{x}\,dx = \int_{u=\ln 1=0}^{u=\ln 4} \frac{u^3}{x}\cdot xdu = \int_{u=0}^{u=\ln 4} u^3\,du = \left[\frac{1}{4}u^4\right]_{u=0}^{u=\ln 4}$$

$$= \left[\frac{1}{4}(\ln 4)^4 - \frac{1}{4}0^4\right] = \frac{1}{4}(\ln 4)^4$$

5. Evaluate each of the following definite integrals.

(e) 5 分

$$\frac{d}{dx}\left[\int_{\sqrt{x}}^{x} \frac{\cos t}{t}\,dt\right] = \frac{d}{dx}\left[\int_{1}^{x} \frac{\cos t}{t}\,dt + \int_{\sqrt{x}}^{1} \frac{\cos t}{t}\,dt\right]$$

$$= \left[\frac{\cos x}{x} - 0\right] + \left[0 - \frac{\cos\sqrt{x}}{\sqrt{x}}\cdot\frac{d}{dx}[\sqrt{x}]\right]$$

$$= \frac{\cos x}{x} - 0 + 0 - \frac{cos\sqrt{x}}{\sqrt{x}}\cdot\frac{1}{2}x^{-\frac{1}{2}} = \frac{\cos x}{x} - \frac{\cos\sqrt{x}}{\sqrt{x}}\cdot\frac{1}{2\sqrt{x}}$$