國 立 雲 林 科 技 大 學

112 學年度第二學期工學院會考試題

時間：10:10~12:00
科目：微積分
班級：

※本次測驗禁止使用電子計算機

1. 求下列積分： $(6\% \times 10 = 60\%)$

(a) $\int e^{-x}\tan(e^{-x})\,dx$　　(b) $\int \frac{1}{x\sqrt{1-(\ln x)^2}}\,dx$　　(c) $\int x^2\cos 4x\,dx$

(d) $\int 2x\arctan x\,dx$　　(e) $\int \frac{\cos^3 x}{\sqrt{\sin x}}\,dx$　　(f) $\int \tan^3\frac{\pi x}{2}\sec\frac{\pi x}{2}\,dx$

(g) $\int \frac{1}{(16-x^2)^{3/2}}\,dx$　　(h) $\int \frac{\sqrt{x^2-16}}{x^2}\,dx$　　(i) $\int \frac{x+1}{x(x^2+1)}\,dx$

(j) $\int_{-\infty}^{\infty} \frac{4}{16+x^2}\,dx$

2. 求以下兩條曲線所包圍的面積。 (10%)

$y = x^2,\quad y = 4x - x^2$

3. 求以下兩條曲線所包圍的區域繞 x 軸旋轉所得旋轉體的體積。 (15%)

$y = x^3,\quad y = \sqrt{x}$

4. 求以下曲線的弧長。 (15%)

$36y^2 = (x^2-4)^3,\quad 1 \le x \le 3,\quad y \ge 0$

《試題完》

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112 學年度第二學期工學院會考試題

時間：10:10~12:00
科目：Calculus
班級：

※ The use of scientific calculators is not permitted in examinations.

1. Find the integral. (6% × 10 = 60%)

(a) $\int e^{-x}\tan(e^{-x})\,dx$ (b) $\int \frac{1}{x\sqrt{1-(\ln x)^2}}\,dx$ (c) $\int x^2\cos 4x\,dx$

(d) $\int 2x\arctan x\,dx$ (e) $\int \frac{\cos^3 x}{\sqrt{\sin x}}\,dx$ (f) $\int \tan^3\frac{\pi x}{2}\sec\frac{\pi x}{2}\,dx$

(g) $\int \frac{1}{(16-x^2)^{3/2}}\,dx$ (h) $\int \frac{\sqrt{x^2-16}}{x^2}\,dx$ (i) $\int \frac{x+1}{x(x^2+1)}\,dx$

(j) $\int_{-\infty}^{\infty} \frac{4}{16+x^2}\,dx$

2. Find the area enclosed by the given curves. (10%)

$y=x^2,\quad y=4x-x^2$

3. Find the volume of the solid obtained by rotating the region bounded by the given curves about the *x*-axis. (15%)

$y=x^3,\quad y=\sqrt{x}$

4. Find the exact length of the curve. (15%)

$36y^2=(x^2-4)^3,\quad 1\le x\le 3,\quad y\ge 0$

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標準答案試算表

No. 1

1.

(a) $\int e^{-x}\tan(e^{-x})\,dx = \int \frac{-\sin(e^{-x})}{\cos(e^{-x})}(-e^{-x})dx = \ln\left|\cos(e^{-x})\right| + C$

(b) $\int \frac{1}{x\sqrt{1-(\ln x)^2}}dx = \int \frac{1}{\sqrt{1-(\ln x)^2}}\frac{1}{x}dx = \arcsin(\ln x) + C$

(c) Let $dv = \cos 4x dx$ and $u = x^2 \rightarrow v = \frac{1}{4}\sin 4x$ and $du = 2x\,dx$

$$\int x^2\cos 4x\,dx = \frac{1}{4}x^2\sin 4x - \frac{1}{2}\int x\sin 4x\,dx \qquad (1)$$

Let $dv = \sin 4x dx$ and $u = x \rightarrow v = -\frac{1}{4}\cos 4x$ and $du = dx$

$$\int x\sin 4x\,dx = -\frac{1}{4}x\cos 4x + \frac{1}{4}\int \cos 4x\,dx = -\frac{1}{4}x\cos 4x + \frac{1}{16}\sin 4x \qquad (2)$$

Substitute equation (2) into equation (1).

$$\int x^2\cos 4x\,dx = \frac{1}{4}x^2\sin 4x + \frac{1}{8}x\cos 4x - \frac{1}{32}\sin 4x + C$$

(d) Let $dv = 2x dx$ and $u = \arctan x \rightarrow v = x^2$ and $du = \frac{1}{1+x^2}dx$

$$\int 2x\arctan x\,dx = x^2\arctan x - \int \frac{x^2}{1+x^2}dx = x^2\arctan x - \int\left(1 - \frac{1}{1+x^2}\right)dx$$

$$= x^2\arctan x - (x - \arctan x) + C = x^2\arctan x - x + \arctan x + C$$

(e) Let $u = \sin x$, $du = \cos x\,dx$

$$\int \frac{\cos^3 x}{\sqrt{\sin x}}dx = \int \frac{(1-\sin^2 x)}{\sqrt{\sin x}}\cos x dx = \int \frac{(1-u^2)}{u^{1/2}}du = \int (u^{-1/2} - u^{3/2})\,du$$

$$= 2u^{1/2} - \frac{2}{5}u^{5/2} + C = 2\sqrt{\sin x} - \frac{2}{5}(\sin x)^{5/2} + C$$

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標準答案試算表

(f) $\int \tan^3\frac{\pi x}{2}\sec\frac{\pi x}{2}dx = \frac{2}{\pi}\int\left(\sec^2\frac{\pi x}{2}-1\right)\left(\frac{\pi}{2}\sec\frac{\pi x}{2}\tan\frac{\pi x}{2}\right)dx$

$= \frac{2}{3\pi}\sec^3\frac{\pi x}{2}-\frac{2}{\pi}\sec\frac{\pi x}{2}+C$

(g) Let $x=4\sin\theta$. Then, $dx=4\cos\theta\,d\theta$ and $\sqrt{16-x^2}=4\cos\theta$

$\int\frac{1}{(16-x^2)^{3/2}}dx = \int\frac{1}{(4\cos\theta)^3}4\cos\theta\,d\theta = \frac{1}{16}\int\frac{1}{\cos^2\theta}d\theta$

$= \frac{1}{16}\int\sec^2\theta\,d\theta = \frac{1}{16}\tan\theta+C = \frac{x}{16\sqrt{16-x^2}}+C$

(h) Let $x=4\sec\theta$. Then, $dx=4\sec\theta\tan\theta\,d\theta$ and $\sqrt{x^2-16}=4\tan\theta$

$\int\frac{\sqrt{x^2-16}}{x^2}dx = \int\frac{4\tan\theta}{16\sec^2\theta}4\sec\theta\tan\theta\,d\theta = \int\frac{\tan^2\theta}{\sec\theta}d\theta = \int\frac{\sec^2\theta-1}{\sec\theta}d\theta$

$= \int(\sec\theta-\cos\theta)d\theta = \ln\left|\sec\theta+\tan\theta\right|-\sin\theta+C$

$= \ln\left|\frac{x}{4}+\frac{\sqrt{x^2-16}}{4}\right|-\frac{\sqrt{x^2-16}}{x}+C$

(i) $\frac{x+1}{x(x^2+1)} = \frac{A}{x}+\frac{Bx+C}{x^2+1} = \frac{1}{x}+\frac{-x+1}{x^2+1}$

$\int\frac{x+1}{x(x^2+1)}dx = \int\left(\frac{1}{x}+\frac{-x+1}{x^2+1}\right)dx = \int\left(\frac{1}{x}-\frac{x}{x^2+1}+\frac{1}{x^2+1}\right)dx$

$= \ln|x|-\frac{1}{2}\ln(x^2+1)+\arctan x+C$

(j) $\int_{-\infty}^{\infty} \frac{4}{16+x^2}\,dx = \lim_{a\to-\infty}\int_{a}^{4} \frac{4}{4^2+x^2}\,dx + \lim_{b\to\infty}\int_{4}^{\infty} \frac{4}{4^2+x^2}\,dx$

$\int_{-\infty}^{\infty} \frac{4}{16+x^2}\,dx = \lim_{a\to-\infty}\left[\arctan\frac{x}{4}\right]_{a}^{4} + \lim_{b\to\infty}\left[\arctan\frac{x}{4}\right]_{4}^{b}$

$= \frac{\pi}{4} - \left(-\frac{\pi}{2}\right) + \frac{\pi}{2} - \frac{\pi}{4} = \pi$

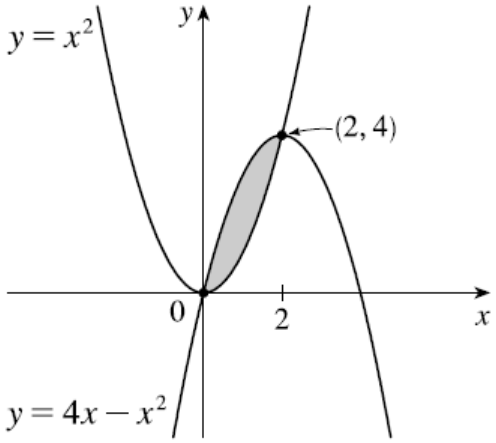
2. 求以下兩條曲線所包圍的面積。 (10%)

$y = x^2, \quad y = 4x - x^2$

The curves intersect at the points (0, 0) and (2, 4).

$A = \int_0^2 \left[(4x - x^2) - x^2\right] dx = \int_0^2 (4x - 2x^2)\,dx$

$= \left[2x^2 - \frac{2}{3}x^3\right]_0^2 = \left[\left(8 - \frac{16}{3}\right) - 0\right] = \frac{8}{3}$



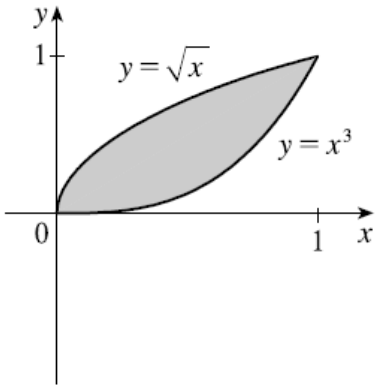
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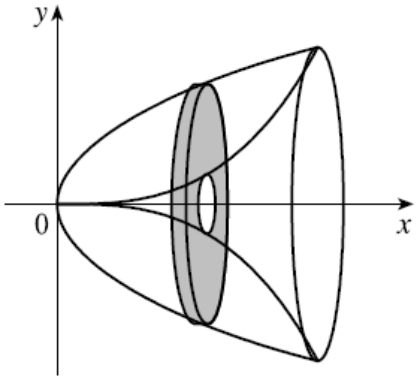
標準答案試算表

3. 求以下兩條曲線所包圍的區域繞 x 軸旋轉所得旋轉體的體積。 (15%)

$y = x^3, \quad y = \sqrt{x}$

The curves intersect at the points (0, 0) and (1, 1).

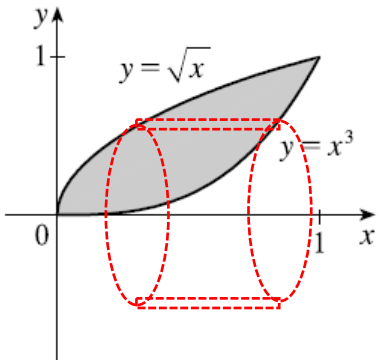




$$A(x) = \pi(\sqrt{x})^2 - \pi(x^3)^2 = \pi(x - x^6)$$

$$V = \int_0^1 \pi(x - x^6)\,dx = \pi\left[\frac{1}{2}x^2 - \frac{1}{7}x^7\right]_0^1 = \pi\left[\left(\frac{1}{2} - \frac{1}{7}\right) - 0\right] = \frac{5}{14}\pi$$

Or using the method of cylindrical shells:



$$V = \int_0^1 2\pi y\,(y^{1/3} - y^2)\,dy = 2\pi\int_0^1 (y^{4/3} - y^3)\,dy$$

$$= 2\pi\left[\frac{3}{7}y^{7/3} - \frac{1}{4}x^4\right]_0^1 = 2\pi\left[\left(\frac{3}{7} - \frac{1}{4}\right) - 0\right]$$

$$= 2\pi \cdot \frac{5}{28} = \frac{5}{14}\pi$$

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標準答案試算表

No. 5

4. 求以下曲線的弧長。 (15%)

$36y^2=(x^2-4)^3,\quad 1\le x\le 3,\quad y\ge 0$

$36y^2=(x^2-4)^3,\quad 1\le x\le 3,\quad y\ge 0$

$\Rightarrow\ y=\dfrac{1}{6}(x^2-4)^{3/2}\ \Rightarrow\ \dfrac{dy}{dx}=\dfrac{1}{6}\cdot\dfrac{3}{2}(x^2-4)^{1/2}\cdot 2x=\dfrac{1}{2}x\,(x^2-4)^{1/2}$

$\Rightarrow\ 1+\left(\dfrac{dy}{dx}\right)^2=1+\dfrac{1}{4}x^2\,(x^2-4)=1-x^2+\dfrac{1}{4}x^4=\dfrac{1}{4}(x^4-4x^2+4)=\dfrac{1}{4}(x^2-2)^2$

$\Rightarrow\ L=\int_1^3\sqrt{\dfrac{1}{4}(x^2-2)^2}\,dx=\int_1^3\dfrac{1}{2}(x^2-2)\,dx=\left[\dfrac{1}{2}\dfrac{1}{3}x^3-x\right]_1^3=\left[\left(\dfrac{27}{6}-3\right)-\left(\dfrac{1}{6}-1\right)\right]$

$=\dfrac{9}{6}+\dfrac{5}{6}=\dfrac{14}{6}=\dfrac{7}{3}$