國 立 雲 林 科 技 大 學

112 學年度第一學期工程學院會考試題

時間：2023/01/10 10:10~12:00
科目：微積分(一)
班級：

❖ 本次會考**禁止使用計算機。**

1. 函數$f(x)=\sin^2 x-\cos x$，求$f(x)$在區間$[0,\pi]$上的絕對極大值(absolute maximum)與絕對極小值(absolute minimum)。(10%)

2. 函數$f(x)=\frac{3}{5}x^{\frac{5}{3}}-3x^{\frac{2}{3}}$，求

   (a) $f(x)$為遞增(increasing)與遞減(decreasing)的區間。(10%)

   (b) $f(x)$的反曲點(inflection points)。(10%)

3. 求極限或證明極限不存在。(10%)

$$\lim_{x\to-\infty}\frac{\sqrt{16x^{10}+8x^5-16}}{6x^5-6x^3+16}$$

4. 求值 (10%)。

$$\lim_{x\to 0}\frac{-10x^2}{1-\sec^2 x}$$

5. 試計算下列函數之定積分。

$(a)\int_3^0(15x^4-13x^2+x)\,dx$ (6%)　$(b)\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{x^2(tanx)}{1+cosx}dx$ (6%)　$(c)\int_1^4\sqrt{w}\,e^{1-\sqrt{w^3}}dw$ (6%)

6. 試求下列函數對 x 的導數。

$$\int_7^{\sin(6x)}\sqrt{t^2+4}\,dt \quad (7\%)$$

7. 試求下列函數之不定積分。

$(a)\int\frac{4w+3}{4w^2+6w-1}dw$ (6%)　$(b)\int(7y-2y^3)\,e^{y^4-7y^2}dy$ (6%)

$(c)\int x^3\sqrt{1+2x^2}\,dx$ (6%)　$(d)\int tan^3 x\,dx$ (7%)

Hint: $tan^2x=sec^2x-1$

❖ The use of scientific calculators is not permitted in examinations.

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112 學年度第一學期工程學院會考試題

時間：2023/01/10 10:10~12:00

科目：Calculus (I)

班級：

1. Find the absolute maximum and the absolute minimum values on the given interval $[0, \pi]$. (10%)

$$f(x) = \sin^2 x - \cos x$$

2. $f(x) = \frac{3}{5}x^{\frac{5}{3}} - 3x^{\frac{2}{3}}$

(a) Find the increasing intervals and the decreasing intervals (10%)

(b) Find the inflection points (10%)

3. Find the limit of show that it does not exist (10%)

$$\lim_{x \to -\infty} \frac{\sqrt{16x^{10} + 8x^5 - 16}}{6x^5 - 6x^3 + 16}$$

4. Evaluate (10%)

$$\lim_{x \to 0} \frac{-10x^2}{1 - \sec^2 x}$$

5. Evaluate each of the following definite integrals.

$(a) \int_3^0 (15x^4 - 13x^2 + x)\, dx$ (6%) $(b) \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{x^2(tanx)}{1 + cosx} dx$ (6%) $(c) \int_1^4 \sqrt{w}\, e^{1-\sqrt{w^3}} dw$ (6%)

6. Differentiate the following integral with respect to x.

$$\int_7^{\sin(6x)} \sqrt{t^2 + 4}\, dt \quad (7\%)$$

7. Evaluate each of the following indefinite integrals.

$(a) \int \frac{4w + 3}{4w^2 + 6w - 1} dw$ (6%) $(b) \int (7y - 2y^3)\, e^{y^4 - 7y^2} dy$ (6%)

$(c) \int x^3 \sqrt{1 + 2x^2}\, dx$ (6%) $(d) \int tan^3 x\, dx$ (7%)

Hint: $tan^2 x = sec^2 x - 1$

1. Find the absolute maximum and the absolute minimum values on the given interval $[0,\pi]$.
$f(x)=\sin^2(x)-\cos(x)$ (10 分)

Sol:

$f'(x)=2\sin(x)\cdot\cos(x)-[-\sin(x)]=2\sin(x)\cos(x)+\sin(x)=\sin(x)[2\cos(x)+1]$

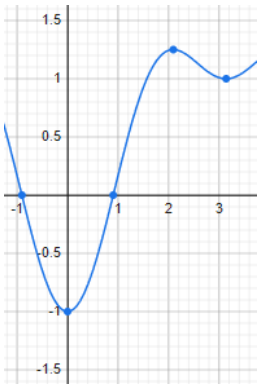
Set $f'(x)=\sin(x)[2\cos(x)+1]=0 \Rightarrow \sin(x)=0, \cos(x)=-\frac{1}{2}$

So, the critical numbers are $x=0, x=\frac{2}{3}\pi$ on $[0,\pi]$

$f(0)=\sin^2(0)-\cos(0)=0^2-1=-1$, the absolute minimum

$f\left(\frac{2}{3}\pi\right)=\sin^2\left(\frac{2}{3}\pi\right)-\cos\left(\frac{2}{3}\pi\right)=\left(\frac{\sqrt{3}}{2}\right)^2-\left(-\frac{1}{2}\right)=\frac{3}{4}+\frac{1}{2}=\frac{5}{4}=1.25$, the absolute maximum

$f(\pi)=\sin^2(\pi)-\cos(\pi)=0^2-(-1)=1$



2. (a) $f(x)=\frac{3}{5}x^{\frac{5}{3}}-3x^{\frac{2}{3}}$, find the increasing intervals and the decreasing intervals (10 分)

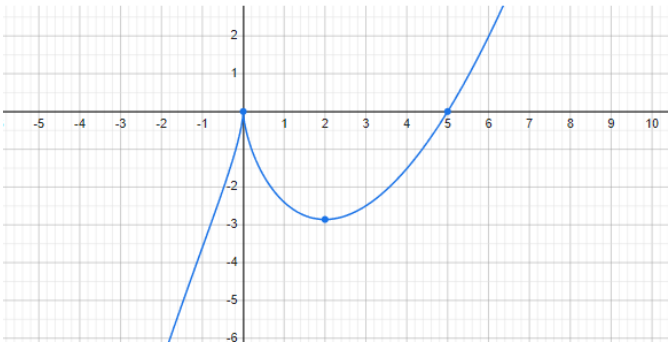
Sol:

$$f'(x)=\frac{3}{5}\cdot\frac{5}{3}x^{\frac{5}{3}-1}-3\cdot\frac{2}{3}x^{\frac{2}{3}-1}=x^{\frac{2}{3}}-2x^{-\frac{1}{3}}=x^{-\frac{1}{3}}(x-2)$$

Set $f'(x)=x^{-\frac{1}{3}}(x-2)=0$, the critical numbers are $x=0$ and $x=2$

The increasing intervals are $(-\infty,0)\cup(2,\infty)$

The decreasing interval is $(0,2)$



2. (b) Find the inflection points (10 分)

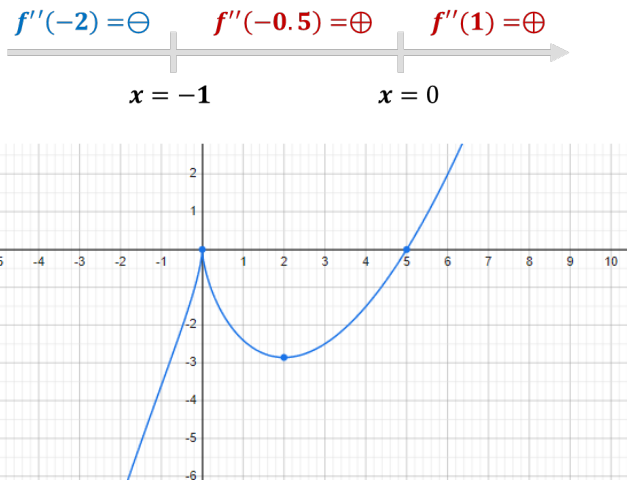
Sol:

$$f''(x)=\frac{2}{3}x^{\frac{2}{3}-1}-2\cdot\left(-\frac{1}{3}\right)x^{-\frac{1}{3}-1}=\frac{2}{3}x^{-\frac{1}{3}}+\frac{2}{3}x^{-\frac{4}{3}}=\frac{2}{3}x^{-\frac{4}{3}}(x+1)$$

Set $f''(x)=\frac{2}{3}x^{-\frac{4}{3}}(x+1)=0$, the possible inflection points are $x=0$ and $x=-1$

But actually only $x=-1$ is the inflection point according to the second derivative test

Therefore, the inflection point is $(-1,-2.4)$



3. Find the limit or show that it does not exist (10 分)

$$\lim_{x\to-\infty}\frac{\sqrt{16x^{10}+8x^5-16}}{6x^5-6x^3+16}$$

Sol:

$$\lim_{x\to-\infty}\frac{\sqrt{16x^{10}+8x^5-16}}{6x^5-6x^3+16}=\lim_{x\to-\infty}\frac{\frac{\sqrt{16x^{10}+8x^5-16}}{x^5}}{\frac{6x^5}{x^5}-\frac{6x^3}{x^5}+\frac{16}{x^5}}=\lim_{x\to-\infty}\frac{\frac{\sqrt{16x^{10}+8x^5-16}}{\sqrt{x^{10}}}}{\frac{6x^5}{|x^5|}-\frac{6x^3}{|x^5|}+\frac{16}{|x^5|}}$$

$$=\lim_{x\to-\infty}\frac{\sqrt{\frac{16x^{10}}{x^{10}}+\frac{8x^5}{x^{10}}-\frac{16}{x^{10}}}}{\frac{6x^5}{-x^5}-\frac{6x^3}{-x^5}+\frac{16}{-x^5}}=\lim_{x\to-\infty}\frac{\sqrt{16+\frac{8}{x^5}-\frac{16}{x^{10}}}}{-6+\frac{6}{x^2}-\frac{16}{x^5}}=\frac{\sqrt{16+0-0}}{-6+0-0}=\frac{4}{-6}=-\frac{2}{3}$$

4. Evaluate $\lim_{x\to 0}\frac{-10x^2}{1-\sec^2(x)}$ (10 分)

Sol:

$$\lim_{x\to 0}\frac{-10x^2}{1-\sec^2(x)}=\frac{-10\cdot 0^2}{1-1^2}=\frac{0}{0}\xrightarrow{L'Hopital}\lim_{x\to 0}\frac{-10\cdot 2x}{-2\sec(x)\cdot\sec(x)\tan(x)}$$
$$=\lim_{x\to 0}\frac{-20x}{-2\sec^2(x)\tan(x)}=\frac{-20\cdot 0}{-2\cdot 1^2\cdot 0}$$
$$=\frac{0}{0}\xrightarrow{L'Hopital}\lim_{x\to 0}\frac{-20\cdot 1}{-2\cdot 2\sec(x)\cdot\sec(x)\tan(x)\cdot\tan(x)-2\sec^2(x)\sec^2(x)}$$
$$=\lim_{x\to 0}\frac{-20}{-4\sec^2(x)\tan^2(x)-2\sec^4(x)}=\frac{-20}{-4\cdot 1^2\cdot 0^2-2\cdot 1^2}=\frac{-20}{0-2}=10$$

5. (a)

$\int_3^0(15x^4-13x^2+x)\,dx$ (6 分)

Sol:

$$\int_3^0(15x^4-13x^2+x)\,dx=(3x^5-\frac{13}{3}x^3+\frac{1}{2}x^2)|_3^0=0-\frac{1233}{2}=-\frac{1233}{2}$$

5. (b)

$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{x^2(tanx)}{1+cosx}dx$ (6 分)

Sol:

$\frac{x^2(tanx)}{1+cosx}$ $is\ an\ odd\ function\ \rightarrow\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\frac{x^2(tanx)}{1+cosx}dx=0.$

5. (c)

$\int_1^4\sqrt{w}\,e^{1-\sqrt{w^3}}dw$ (6 分)

Sol:

Set $u=1-w^{\frac{3}{2}}$

$du=-\frac{3}{2}w^{\frac{1}{2}}dw\ \rightarrow\ \sqrt{w}\,dw=-\frac{2}{3}du$

$$\int_1^4\sqrt{w}\,e^{1-\sqrt{w^3}}dw=-\frac{2}{3}\int_0^{-7}e^u\,du=-\frac{2}{3}e^u|_0^{-7}=\frac{2}{3}(1-e^{-7})$$

6. Differentiate the following integral with respect to x. (本題 7 分)

$$\int_7^{\sin(6x)} \sqrt{t^2+4}\,dt$$

Sol:

This is nothing more than a quick application of the Fundamental Theorem of Calculus, Part I. Note however, that because the upper limit is not just x we'll need to use the Chain Rule, with the "inner function" as $sin(6x)$. The derivative is

$$\frac{d}{dx}\int_7^{\sin(6x)} \sqrt{t^2+4}\,dt = 6\cos(6x)\sqrt{sin^2(6x)+4}$$

7. (a)

$$\int \frac{4w+3}{4w^2+6w-1}dw \quad (6\text{ 分})$$

Sol:

Set $u = 4w^2+6w-1$

$$du = (8w+6)dw \;\rightarrow\; (4w+3)dw = \frac{1}{2}du$$

$$\int \frac{4w+3}{4w^2+6w-1}dw = \frac{1}{2}\int \frac{1}{u}du = \frac{1}{2}ln|u| + c$$
$$= \frac{1}{2}ln|4w^2+6w-1| + c$$

7. (b)

$$\int (7y-2y^3)\,e^{y^4-7y^2}dy \quad (6\text{ 分})$$

Sol:

Set $u = y^4-7y^2$

$$du = (4y^3-14y)dy \;\rightarrow\; (7y-2y^3)dy = -\frac{1}{2}du$$

$$\int (7y-2y^3)\,e^{y^4-7y^2}dy = -\frac{1}{2}\int e^u du = -\frac{1}{2}e^u + c$$
$$= -\frac{1}{2}e^{y^4-7y^2} + c$$

7. (c)

$\int x^3\sqrt{1+2x^2}\,dx$ (6 分)

Sol:

Set $u=\sqrt{1+2x^2} \rightarrow u^2=1+2x^2 \rightarrow 2udu=4xdx$

$\rightarrow xdx=\frac{1}{2}udu$

$$\int x^3\sqrt{1+2x^2}\,dx=\int x^2\cdot x\cdot\sqrt{1+2x^2}dx$$

$$=\int\frac{u^2-1}{2}\cdot u\cdot\frac{1}{2}udu$$

$$=\frac{1}{4}\int(u^4-u^2)du=\frac{1}{4}\left(\frac{1}{5}u^5-\frac{1}{3}u^3\right)+C$$

$$=\frac{1}{20}(1+2x^2)^{\frac{5}{2}}-\frac{1}{12}(1+2x^2)^{\frac{3}{2}}+C$$

7. (d)

$\int tan^3 x\,dx$ (7 分)

Sol:

$$\int tan^3 x\,dx=\int tan^2x\cdot tanxdx=\int(sec^2x-1)\cdot tanxdx$$

$$=\int sec^2x\cdot tanxdx-\int tanxdx$$

Set $u=tanx \rightarrow du=sec^2xdx$

$$\int sec^2x\cdot tanxdx=\int udu=\frac{1}{2}tan^2x+c_1\ldots(1)$$

Set $u=cosx \rightarrow du=-sinxdx$

$$\int tanxdx=\int\frac{sinx}{cosx}dx=-\int\frac{1}{u}du=-ln|cosx|+c_2\ldots(2)$$

$$\therefore\int tan^3 x\,dx=\frac{1}{2}tan^2x+ln|cosx|+C$$