國 立 雲 林 科 技 大 學

111 學年度第一學期工學院會考試題

時間：10:10~12:00
科目：微積分
班級：

※本次測驗禁止使用電子計算機

1. 函數 $f(x)=2\cos x+\sin 2x$，求 $f(x)$ 在區間 $[0,\ \pi/2]$ 上的絕對極大值(absolute maximum)與絕對極小值(absolute minimum)。 (10%)

2. 函數 $f(x)=x^{1/3}(x+4)$，求：

   (a) $f(x)$ 為遞增(increasing)或遞減(decreasing)的區間。 (10%)

   (b) $f(x)$ 的相對極大值(local maximum)與相對極小值(local minimum)。 (10%)

   (c) $f(x)$ 的圖形為凹向上(concave up)或凹向下(concave down)的區間及反曲點(inflection points)。 (15%)

3. 求 $\lim_{x\to 1}(\csc \pi x)\ln x$。 (5%)

4. 求下列不定積分： (6% × 6 = 36%)

   (a) $\int\left(\frac{2}{x}-5\cos x+2\sec x\tan x\right)dx$ (b) $\int\frac{1}{t^2}\sqrt{\left(1-\frac{2}{t}\right)^3}\,dt$

   (c) $\int e^{2x}(e^{2x}+1)^{10}\,dx$ (d) $\int\frac{(1+\sin\theta)\cos\theta}{\sqrt{1-\sin\theta}}\,d\theta$

   (e) $\int\frac{1-\sqrt{x}}{\sqrt{x}\left(1+\sqrt{x}\right)^2}dx$ (f) $\int\frac{\sin(\ln x^2)}{x}dx$

5. 求下列定積分： (7% × 2 = 14%)

   (a) $\int_0^1(x+1)\sqrt{1-x}\,dx$ (b) $\int_0^{\frac{\pi}{4}}\frac{1-\sin\theta}{\cos^2\theta}\,d\theta$

《試題完》

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111 學年度第一學期工學院會考試題

時間：10:10~12:00
科目：Calculus
班級：

※ The use of scientific calculators is not permitted in examinations.

1. Find the absolute maximum and absolute minimum values of the function on the given interval. (10%)

$f(x) = 2\cos x + \sin 2x, \quad [0, \pi/2]$

2. (a) Find the intervals of increase or decrease. (10%)
(b) Find the local maximum and minimum values. (10%)
(c) Find the intervals of concavity and the inflection points. (15%)

$f(x) = x^{1/3}(x+4)$

3. Evaluate $\lim_{x\to 1}(\csc \pi x)\ln x$. (5%)

4. Find the indefinite integral. (6% × 6 = 36%)

(a) $\int \left(\frac{2}{x} - 5\cos x + 2\sec x \tan x\right) dx$ (b) $\int \frac{1}{t^2}\sqrt{\left(1-\frac{2}{t}\right)^3}\, dt$

(c) $\int e^{2x}(e^{2x}+1)^{10}\, dx$ (d) $\int \frac{(1+\sin\theta)\cos\theta}{\sqrt{1-\sin\theta}}\, d\theta$

(e) $\int \frac{1-\sqrt{x}}{\sqrt{x}\left(1+\sqrt{x}\right)^2} dx$ (f) $\int \frac{\sin(\ln x^2)}{x} dx$

5. Evaluate the definite integral. (7% × 2 = 14%)

(a) $\int_0^1 (x+1)\sqrt{1-x}\, dx$ (b) $\int_0^{\frac{\pi}{4}} \frac{1-\sin\theta}{\cos^2\theta}\, d\theta$

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標準答案試算表

No. 1

1.

$f(x) = 2\cos x + \sin 2x, \quad [0, \pi/2]$

$f'(x) = -2\sin x + 2\cos 2x = -2\sin x + 2(1 - 2\sin^2 x)$

$= -2(2\sin^2 x + \sin x - 1) = -2(2\sin x - 1)(\sin x + 1)$

$f(x) = 0$: $\sin x = 1/2$ or -1. Only $x = \pi/6$ is in the domain of $[0, \pi/2]$.

$f(0) = 2; \quad f(\pi/6) = \sqrt{3} + \frac{1}{2}\sqrt{3} = \frac{3}{2}\sqrt{3}; \quad f(\pi/2) = 0$

So $f(\pi/6) = \frac{3}{2}\sqrt{3}$ is the absolute maximum value, and

$f(\pi/2) = 0$ is the absolute maximum value.

2.

(a) $f(x) = x^{1/3}(x+4) = x^{4/3} + 4x^{1/3}$

$f'(x) = \frac{4}{3}x^{1/3} + \frac{4}{3}x^{-2/3} = \frac{4}{3}x^{-2/3}(x+1) = \frac{4(x+1)}{3\sqrt[3]{x^2}}$

$f'(x) > 0$ if $-1 < x < 0$ or $x > 0$

$f'(x) < 0$ if $x < -1$

f is increasing on $(-1, \infty)$ and decreasing on $(-\infty, -1)$,

(b) f changes from decreasing to increasing at $x = -1$, so $f(-1) = -3$ is a local minimum value. (There is no local maximum value.)

(c) $f''(x) = \frac{4}{9}x^{-2/3} - \frac{8}{9}x^{-5/3} = \frac{4}{9}x^{-5/3}(x-2) = \frac{4(x-2)}{9\sqrt[3]{x^5}}$

$f''(x) > 0$ when $x > 2$ and $x < 0 \Rightarrow$ concave upward on $(-\infty, 0)$ and $(2, \infty)$.

$f''(x) < 0$ when $0 < x < 2 \Rightarrow$ concave downward on $(0, 2)$

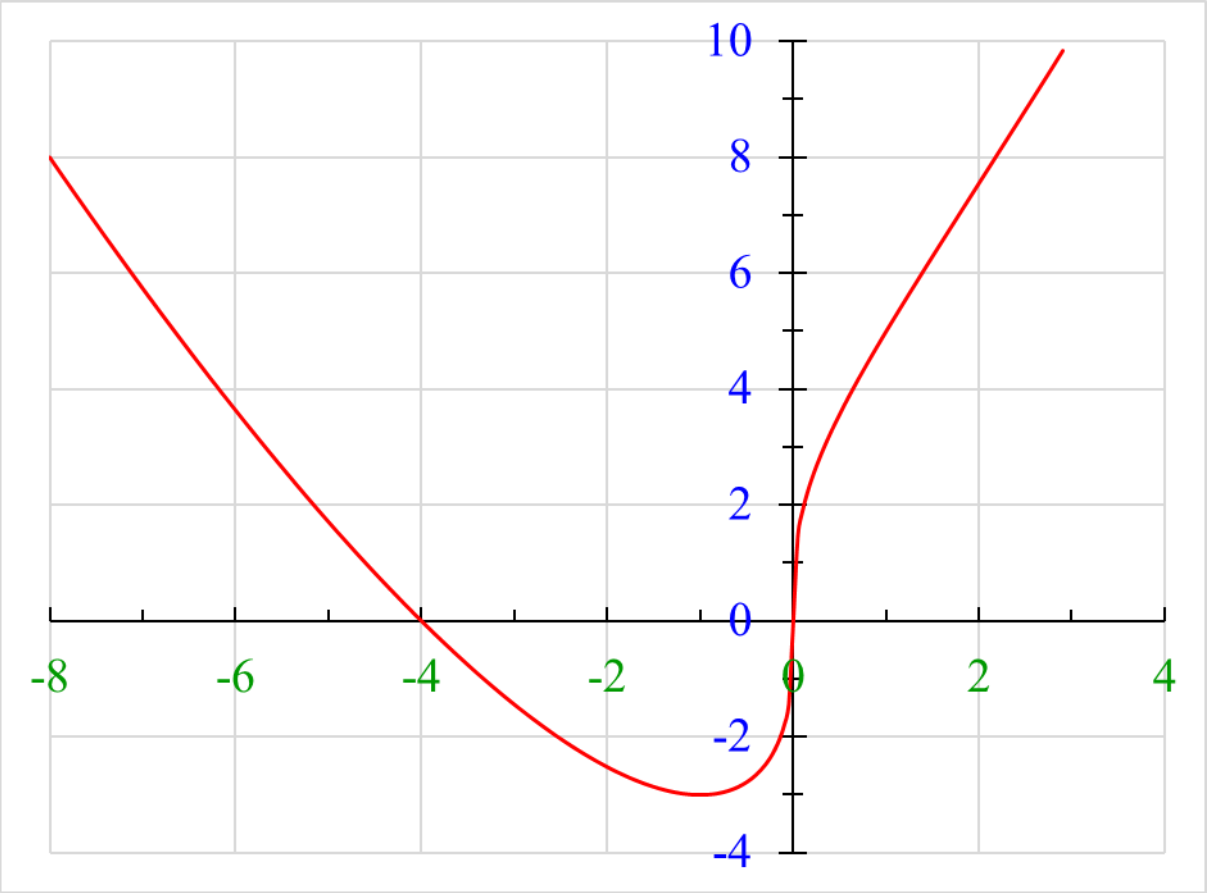
Inflection points: $(0, 0)$ and $(2, 6\sqrt[3]{2})$.

(下圖為第 2 題的函數曲線，僅供閱卷時參考，學生作答時不需畫圖)

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標準答案試算表

No. 2



3.

$$\lim_{x\to 1}(\csc \pi x)\ln x = \lim_{x\to 1}\frac{\ln x}{\sin \pi x} = \lim_{x\to 1}\frac{1/x}{\pi \cos \pi x} = -\frac{1}{\pi}$$

4.

(a)

$$\int \left(\frac{2}{x} - 5\cos x + 2\sec x \tan x\right) dx = 2\ln|x| - 5\sin x + 2\sec x + C$$

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標準答案試算表

(b) Let $u=1-\frac{2}{t}$, $du=\frac{2}{t^2}dt$

$$\int \frac{1}{t^2}\sqrt{\left(1-\frac{2}{t}\right)^3}\,dt = \frac{1}{2}\int u^{3/2}du = \frac{1}{2}\cdot\frac{2}{5}u^{5/2}+C = \frac{1}{5}\sqrt{\left(1-\frac{2}{t}\right)^5}+C$$

(c)

$$\int e^{2x}(e^{2x}+1)^{10}\,dx = \frac{1}{2}\int (e^{2x}+1)^{10}\,2e^{2x}dx = \frac{1}{2}\cdot\frac{(e^{2x}+1)^{11}}{11}+C = \frac{1}{22}(e^{2x}+1)^{11}+C$$

(d) Let $u=1-\sin\theta$, $\sin\theta=1-u$, $\cos\theta\,d\theta=-du$

$$\int \frac{(1+\sin\theta)\cos\theta}{\sqrt{1-\sin\theta}}\,d\theta = \int \frac{2-u}{u^{1/2}}(-du) = \int (u^{1/2}-2u^{-1/2})\,du = \frac{2}{3}u^{3/2}-4u^{1/2}+C$$
$$= \frac{2}{3}(1-\sin\theta)^{3/2}-4(1-\sin\theta)^{1/2}+C$$

(e) Let $u=1+\sqrt{x}$, $\sqrt{x}=u-1$, $du=\frac{1}{2\sqrt{x}}dx$

$$\int \frac{1-\sqrt{x}}{\sqrt{x}\left(1+\sqrt{x}\right)^2}dx = 2\int \frac{2-u}{u^2}\,du = \int\left(4u^{-2}-\frac{2}{u}\right)du = -\frac{4}{u}-2\ln|u|+C$$
$$= -\frac{4}{1+\sqrt{x}}-2\ln\left(1+\sqrt{x}\right)+C$$

(f) Let $u=\ln x^2$, $du=\frac{2x}{x^2}dx=\frac{2}{x}dx$

$$\int \frac{\sin(\ln x^2)}{x}dx = \frac{1}{2}\int \sin u\,du = -\frac{1}{2}\cos u+C = -\frac{1}{2}\cos(\ln x^2)+C$$

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標準答案試算表

5.

(a) Let $u=1-x$, $x=1-u$, $dx=-du$,

$x=0\to u=1,\quad x=1\to u=0$

$$\int_0^1 (x+1)\sqrt{1-x}\,dx=\int_1^0 (2-u)u^{1/2}(-du)=\int_0^1 (2u^{1/2}-u^{3/2})\,du$$

$$=\left[\frac{4}{3}u^{3/2}-\frac{2}{5}u^{5/2}\right]=\frac{4}{3}-\frac{2}{5}=\frac{14}{15}$$

(b)

$$\int_0^{\frac{\pi}{4}}\frac{1-\sin\theta}{\cos^2\theta}\,d\theta=\int_0^{\frac{\pi}{4}}\left(\frac{1}{\cos^2\theta}-\frac{1}{\cos\theta}\cdot\frac{\sin\theta}{\cos\theta}\right)d\theta$$

$$=\int_0^{\frac{\pi}{4}}(\sec^2\theta-\sec\theta\tan\theta)\,d\theta$$

$$=\left[\tan\theta-\sec\theta\right]_0^{\pi/4}$$

$$=\left(1-\sqrt{2}\right)-(0-1)$$

$$=2-\sqrt{2}$$