國 立 雲 林 科 技 大 學

110 學年度第一學期工學院會考試題

時間：10:10~12:00
科目：微積分
班級：

※本次測驗禁止使用電子計算機

1. 函數 $f(x)=2x^3-3x^2-12x+1$，求： (40%)

   (a) $f(x)$在區間 $[-2, 4]$ 上的絕對極大值(absolute maximum)與絕對極小值(absolute minimum)。

   (b) $f(x)$在 $(-\infty, \infty)$ 內的相對極大值(local maximum)與相對極小值(local minimum)。

   (c) $f(x)$為遞增(increasing)的區間及遞減(decreasing)的區間。

   (d) 圖形為凹向上(concave up)的區間及凹向下(concave down)的區間。

2. 求極限或證明極限不存在。 (10%)

$$\lim_{x\to-\infty}\frac{\sqrt{4+4x^6}}{3-x^3}$$

3. 求下列不定積分： (6% × 5 = 30%)

(a) $\int\frac{3x^2-2x+1}{\sqrt{x}}dx$ (b) $\int\frac{e^{\sqrt{x}}\sqrt{1+e^{\sqrt{x}}}}{\sqrt{x}}dx$ (c) $\int\frac{\sin^2 x\cos x}{\sqrt{2+\sin x}}dx$

(d) $\int e^{-2x}\sec(e^{-2x})\tan(e^{-2x})dx$ (e) $\int\frac{(\ln x)^2-2\ln x+3}{x\ln x}dx$

4. 求下列定積分： (7% × 2 = 14%)

(a) $\int_{-1}^{0}x^2\sqrt{x+1}\,dx$ (b) $\int_{0}^{\pi/3}\sqrt{1+\tan^2\theta}\,d\theta$

5. 求 $F(x)=\int_{-x^2-1}^{x^2+1}\ln(t^2+1)dt$ 對 x 的導數 (derivative)。 (6%)

《試題完》

國 立 雲 林 科 技 大 學

110 學年度第一學期工學院會考試題

時間：10:10~12:00
科目：Calculus
班級：

※ The use of scientific calculators is not permitted in examinations.

1. (a) Find the absolute maximum and absolute minimum values on the given interval [−2, 4].
   (b) Find the local maximum and local minimum values of the function.
   (c) Find the intervals of increase or decrease.
   (d) Find the intervals of concavity.

$f(x) = 2x^3 - 3x^2 - 12x + 1$ (40%)

2. Find the limit or show that it does not exist. (10%)

$$\lim_{x \to -\infty} \frac{\sqrt{4 + 4x^6}}{3 - x^3}$$

3. Find the indefinite integral. (6% × 5 = 30%)

(a) $\int \frac{3x^2 - 2x + 1}{\sqrt{x}} dx$ (b) $\int \frac{e^{\sqrt{x}} \sqrt{1 + e^{\sqrt{x}}}}{\sqrt{x}} dx$ (c) $\int \frac{\sin^2 x \cos x}{\sqrt{2 + \sin x}} dx$

(d) $\int e^{-2x} \sec(e^{-2x}) \tan(e^{-2x}) dx$ (e) $\int \frac{(\ln x)^2 - 2\ln x + 3}{x \ln x} dx$

4. Evaluate the definite integral. (7% × 2 = 14%)

(a) $\int_{-1}^{0} x^2 \sqrt{x + 1}\, dx$ (b) $\int_{0}^{\pi/3} \sqrt{1 + \tan^2 \theta}\, d\theta$

5. Find the derivative of $F(x) = \int_{-x^2-1}^{x^2+1} \ln(t^2 + 1)\, dt$ 。 (6%)

國立雲林科技大學 110 學年度第一學期工學院微積分統一會考

標準答案試算表

No. 1

1.

(a) $f'(x) = 6x^2 - 6x - 12 = 6(x^2 - x - 2) = 6(x+1)(x-2)$

$f'(x) = 0 \;\Rightarrow\; x = -1,\ 2$

$f(-2) = -3\ ;\ f(-1) = 8\ ;\ f(2) = -19\ ;\ f(4) = 33$

$\Rightarrow f(4) = 33$ is the absolute maximum value; $f(2) = -19$ is the absolute minimum value.

(b) $f'(x) < 0 \;\Rightarrow\; -1 < x < 2$

$f'(x) > 0 \;\Rightarrow\; x < -1$ or $x > 2$

Since $f'(x)$ changes from positive to negative at $x = -1$, $f(-1) = 8$ is a local maximum value.
Since $f'(x)$ changes from negative to positive at $x = 2$, $f(2) = -19$ is a local minimum value.

(c) From part (b), $f(x)$ is increasing on $(-\infty, -1)$ and $(2, \infty)$, and decreasing on $(-1, 2)$.

(d) $f''(x) = 12x - 6 = 6(2x-1)$

$f''(x) < 0 \;\Rightarrow\; x < 1/2$

$f''(x) > 0 \;\Rightarrow\; x > 1/2$

$\Rightarrow f(x)$ is concave upward on $(1/2, \infty)$ and concave downward on $(-\infty, 1/2)$.

2.

$$\lim_{x\to-\infty} \frac{\sqrt{4+4x^6}}{3-x^3} = \lim_{x\to-\infty} \frac{\sqrt{4+4x^6}\,/\,x^3}{(3-x^3)/x^3} = \frac{\lim_{x\to-\infty} -\sqrt{(4+4x^6)/x^6}}{\lim_{x\to-\infty}(3/x^3 - 1)} = \frac{\lim_{x\to-\infty} -\sqrt{(4/x^6+4)}}{3\lim_{x\to-\infty}(1/x^3) - 1}$$

$$= \frac{-\sqrt{0+4}}{3\cdot 0 - 1} = \frac{-2}{-1} = 2$$

國立雲林科技大學 110 學年度第一學期工學院微積分統一會考

標準答案試算表

3.

(a) $\int \frac{3x^2-2x+1}{\sqrt{x}}dx = \int (3x^{3/2}-2x^{1/2}+x^{-1/2})dx = \frac{6}{5}x^{5/2}-\frac{4}{3}x^{3/2}+2x^{1/2}+C$

(b) Let $u = 1+e^{\sqrt{x}}$ ， $du = \frac{e^{\sqrt{x}}}{2\sqrt{x}}dx$ ，

$$\int \frac{e^{\sqrt{x}}\sqrt{1+e^{\sqrt{x}}}}{\sqrt{x}}dx = \int 2u^{1/2}du = \frac{4}{3}u^{3/2}+C = \frac{4}{3}\left(1+e^{\sqrt{x}}\right)^{3/2}+C$$

(c) Let $u = 2+\sin x$ ， $\sin x = u-2$ ， $\cos x\,dx = du$ ，

$$\int \frac{\sin^2 x\cos x}{\sqrt{2+\sin x}}dx = \int \frac{(u-2)^2}{u^{1/2}}du = \int (u^{3/2}-4u^{1/2}+4u^{-1/2})\,du$$

$$= \frac{2}{5}u^{5/2}-\frac{8}{3}u^{3/2}+8u^{1/2}+C$$

$$= \frac{2}{5}(2+\sin x)^{5/2}-\frac{8}{3}(2+\sin x)^{3/2}+8(2+\sin x)^{1/2}+C$$

(d)

$$\int e^{-2x}\sec(e^{-2x})\tan(e^{-2x})\,dx = -\frac{1}{2}\int \sec(e^{-2x})\tan(e^{-2x})(-2e^{-2x})\,dx$$

$$= -\frac{1}{2}\sec(e^{-2x})+C$$

(e) Let $u = \ln x$ ， $du = \frac{1}{x}dx$ ，

$$\int \frac{(\ln x)^2-2\ln x+3}{x\ln x}dx = \int \frac{u^2-2u+3}{u}du = \int \left(u-2+\frac{3}{u}\right)du$$

$$= \frac{1}{2}u^2-2u+3\ln|u|+C = \frac{1}{2}(\ln x)^2-2\ln x+3\ln|\ln x|+C$$

國立雲林科技大學 110 學年度第一學期工學院微積分統一會考

標準答案試算表

No. 3

4.

(a) Let $u = x+1$ ， $x = u-1$ ， $dx = du$ ，

$$\int_{-1}^{0} x^2\sqrt{x+1}\,dx = \int_0^1 (u-1)^2 u^{1/2}\,du = \int_0^1 (u^{5/2} - 2u^{3/2} + u^{1/2})\,du$$

$$= \left[\frac{2}{7}u^{7/2} - \frac{4}{5}u^{5/2} + \frac{2}{3}u^{3/2}\right]_0^1 = \frac{2}{7} - \frac{4}{5} + \frac{2}{3} = \frac{16}{105}$$

(b)

$$\int_0^{\pi/3} \sqrt{1+\tan^2\theta}\,d\theta = \int_0^{\pi/3} \sqrt{\sec^2\theta}\,d\theta = \int_0^{\pi/3} \sec\theta\,d\theta$$

$$= \left[\ln\left|\sec\theta + \tan\theta\right|\right]_0^{\pi/3} = \ln(2+\sqrt{3}) - \ln(1+0) = \ln(2+\sqrt{3})$$

5.

$$F(x) = \int_{-x^2-1}^{x^2+1} \ln(t^2+1)\,dt = 2\int_0^{x^2+1} \ln(t^2+1)\,dt$$

$$F'(x) = \frac{d}{dx}\left[2\int_0^{x^2+1} \ln(t^2+1)\,dt\right]$$

$$= 2\ln\left[(x^2+1)^2+1\right]\cdot\frac{d}{dx}[x^2+1]$$

$$= 4x\ln\left[x^4+2x^2+2\right]$$