國 立 雲 林 科 技 大 學

109 學年度第 2 學期工程學院會考試題

時間：10:10~12:00

科目：微積分 (Calculus)

班級：

※本次測驗禁止使用電子計算機

1. 求下列積分： (7% × 6 = 42%)

(a) $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \csc^3 x \cot x \, dx$

(b) $\int (e^x + 2x)^2 \, dx$

(c) $\int \frac{\sin^{-1}(3x)}{\sqrt{1-9x^2}} dx$

(d) $\int \frac{e^x}{\sqrt{16-e^{2x}}} dx$

(e) $\int x \ln(x^2) \, dx$

(f) $\int x\sqrt{6x - x^2 - 8} \, dx$

2. 求 $\int \frac{x^3+4x^2-4x-1}{(x^2+1)^2} dx$ 。 (8%)

3. 請計算$f(x) = \sqrt[3]{2x}, g(x) = \frac{1}{8}x^2$在$0 \le x \le 6$範圍內所圍的區域面積。 (10%)

4. 請計算$y = x^2, x = y^2$在$0 \le x \le 1$所圍區域，繞$y = 1$軸旋轉所成旋轉體的體積。 (10%)

5. 請計算$x = y^2, x = 1$在$0 \le x \le 1$所圍區域，繞y軸旋轉所成旋轉體的體積。 (10%)

6. 請計算$f(x) = \ln(\sec x)$在$[0, \frac{\pi}{4}]$區間之弧長。 (10%)

7. 請計算$\int_1^\infty \frac{1}{x^3+x} dx$ 。 (10%)

《試題完》

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1. Find the integral： (7% × 6 = 42%)

(a) $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \csc^3 x \cot x \, dx$

(b) $\int (e^x + 2x)^2 \, dx$

(c) $\int \frac{\sin^{-1}(3x)}{\sqrt{1-9x^2}} dx$

(d) $\int \frac{e^x}{\sqrt{16-e^{2x}}} dx$

(e) $\int x \ln(x^2) \, dx$

(f) $\int x\sqrt{6x - x^2 - 8} \, dx$

2. Find the integral： $\int \frac{x^3+4x^2-4x-1}{(x^2+1)^2} dx$. (8%)

3. Find the area of the region bounded by the curves:

$f(x) = \sqrt[3]{2x}$, $g(x) = \frac{1}{8}x^2$, $0 \le x \le 6$. (10%)

4. Find the volume of the solid obtained by rotating about the $y = 1$ axis the region bounded by $y = x^2, x = y^2, 0 \le x \le 1$. (10%)

5. Find the volume of the solid obtained by rotating about the y-axis the region bounded by $x = y^2, x = 1, 0 \le x \le 1$. (10%)

6. Find the length of the arc of $f(x) = \ln(\sec x)$ between the region $[0, \frac{\pi}{4}]$. (10%)

7. Find the integral: $\int_1^\infty \frac{1}{x^3+x} dx$. (10%)

《試題完》

1.

(a) $\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} csc^3 x \cot x \, dx$

$Let\ u = csc\ x\ ;\ \mathrm{du} = -\csc x \cot x \, dx$

$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} csc^3 x \cot x \, dx$

$= -\int_{\frac{2\sqrt{3}}{3}}^{1} u^2 \, du = \frac{1}{3}[u^3]_{1}^{\frac{2\sqrt{3}}{3}} = \frac{8\sqrt{3}}{27} - \frac{1}{3}$

(b) $\int (e^x + 2x)^2 \, dx$

$= \int (e^{2x} + 4xe^x + 4x^2)^2 \, dx$

$= \frac{1}{2}e^{2x} + \frac{4}{3}x^3 + \int 4xe^x \, dx$

$= \frac{1}{2}e^{2x} + \frac{4}{3}x^3 + 4xe^x - 4\int e^x \, dx$

$= \frac{1}{2}e^{2x} + \frac{4}{3}x^3 + 4xe^x - 4e^x + c$

(c) $\int \frac{\sin^{-1}(3x)}{\sqrt{1-9x^2}} dx$

$Let\ u = sin^{-1}\ 3x\ dx\ ; \mathrm{du} = \frac{3}{\sqrt{1-9x^2}} dx$

$$\int \frac{\sin^{-1}(3x)}{\sqrt{1-9x^2}}\,dx$$

$$= \frac{1}{3}\int u\,du = \frac{1}{6}u^2 + c = \frac{1}{6}(\sin^{-1} 3x)^2 + c$$

(d) $\int \frac{e^x}{\sqrt{16-e^{2x}}}\,dx$

$Let\ u = e^x dx$; du $= e^x\,dx$

$$= \int \frac{1}{\sqrt{16-u^2}}\,du$$

$$= \sin^{-1}(\frac{u}{4}) + c = \sin^{-1}(\frac{e^x}{4}) + c$$

(e) $\int x \ln x^2\,dx$

$Let\ u = x^2$; du $= 2x\,dx$

$$= \frac{1}{2}\int \ln u\,du = \frac{1}{2}u \ln u - \frac{1}{2}u + c$$

$$= \frac{1}{2}x^2 \ln x^2 - \frac{1}{2}x^2 + c$$

(f) $\int x\sqrt{6x - x^2 - 8}\,dx$

$Let\ x - 3 = sin\,u$; dx $=$ cosu du

$$= \int x\sqrt{1-(x-3)^2}\,dx$$

$$= \int (3 + sin\,u)(\text{cosu})\text{cosu}\,du$$

$= \int (3\cos^2 u + \cos^2 u \sin u)\, du$

$= \int [3(\frac{1+\cos 2u}{2}) + \cos^2 u \sin u]\, du$

$= \frac{3u}{2} + \frac{3}{4}\sin 2u - \frac{1}{3}\cos^3 u + c$

$= \frac{3}{2}\sin^{-1}(x-3) + \frac{3}{2}(x-3)\sqrt{6x - x^2 - 8} - \frac{1}{3}(6x - x^2 - 8)^{\frac{3}{2}} + c$

2.

$$\int \frac{x^3+4x^2-4x-1}{(x^2+1)^2}\, dx$$

$$\frac{x^3+4x^2-4x-1}{(x^2+1)^2} = \frac{Ax+B}{x^2+1} + \frac{Cx+D}{(x^2+1)^2}$$

$x^3 + 4x^2 - 4x - 1$

$= (Ax + B)(x^2 + 1) + (Cx + D)$

$A = 1,\ \ B = 4,\ \ C = -5,\ \ D = -5$

$$\int \frac{x^3+4x^2-4x-1}{(x^2+1)^2}\, dx$$

$= \int (\frac{x}{x^2+1} + \frac{4}{x^2+1} - \frac{5x}{(x^2+1)^2} - \frac{5}{(x^2+1)^2})dx$

$$= \frac{1}{2}\ln(x^2+1) + 4\tan^{-1}x + \frac{5}{2(x^2+1)} - 5\int \frac{1}{(x^2+1)^2}\,dx$$

$Let\ x = tan\,u\ ;\ \mathrm{dx} = sec^2u\,du$

$$\int \frac{1}{(x^2+1)^2}\,dx = \int \frac{sec^2u\,du}{(1+tan^2u)^2} = \int cos^2u\,du$$

$$= \frac{1}{2}\int(1+\cos 2u)du = \frac{1}{2}(\tan^{-1}x + \frac{x}{x^2+1)}) + c$$

$$= \frac{1}{2}\ln(x^2+1) + \frac{3}{2}\tan^{-1}x + \frac{5(1-x)}{2(1+x^2)} + c$$

3.

$$\sqrt[3]{2x} = \frac{1}{8}x^2 \Rightarrow 2x = \frac{1}{2^9}x^6$$

$$\Rightarrow x(x^5 - 2^{10}) = 0$$

$$\therefore\ x = 0, \text{or } x = 4$$

$$A = \int_0^4 \left(\sqrt[3]{2x} - \frac{1}{8}x^2\right)dx + \int_4^6 \left(\frac{1}{8}x^2 - \sqrt[3]{2x}\right)dx$$

$$= \left[\frac{3}{4}\sqrt[3]{2}x^{\frac{4}{3}} - \frac{1}{24}x^3\right]|_0^4 + [\frac{1}{24}x^3 - \frac{3}{4}\sqrt[3]{2}x^{\frac{4}{3}}]|_4^6$$

$$= \frac{47}{3} - \frac{9}{2}\sqrt[3]{12}$$

4.

$$V = \int_0^1 \pi\left[(1-x^2)^2 - \left(1-\sqrt{x}\right)^2\right] dx$$

$$= \pi \int_0^1 \left[x^4 - 2x^2 + 2\sqrt{x} - x\right] dx$$

$$= \pi\left[\frac{x^5}{5} - \frac{2}{3}x^3 + \frac{4}{3}x^{3/2} - \frac{x^2}{2}\right]\frac{1}{0} = \frac{11}{30}\pi$$

5.

$$V = \int_{-1}^1 \pi 1^2 - \pi(y^2)^2\, dy$$

$$= 2\pi \int_0^1 (1-y^4)\, dy$$

$$= 2\pi[y - \frac{1}{5}y^5]_0^1 = \frac{8}{5}\,\pi$$

6.

$$f(x) = \ln\sec x\,, \frac{d}{dx} f(x) = \frac{\sec x \tan x}{\sec x} = \tan x$$

$$L = \int_0^{\frac{\pi}{4}} \sqrt{1 + tan^2 x}\, dx = \int_0^{\frac{\pi}{4}} |\sec x|\, dx$$

$$= [\ln|\sec x + tanx|]|_0^{\frac{\pi}{4}} = \ln(\sqrt{2}+1)$$

7.

$$\int_1^{\infty} \frac{1}{(x^3+x)}\,dx$$

$$= \lim_{t\to\infty} \int_1^{t} \frac{1}{x(x^2+1)}\,dx$$

$$= \lim_{t\to\infty} \int_1^{t} [\frac{1}{x} - \frac{x}{x^2+1}]\,dx$$

$$= \lim_{t\to\infty} \left[\ln|x| - \frac{1}{2}\ln|x^2+1|\right] |(x=t) - (x=1)$$

$$= \lim_{t\to\infty} [\ln \frac{x}{\sqrt{x^2+1}}] |(x=t) - (x=1)$$

$$= \lim_{t\to\infty} (\ln \frac{1}{\sqrt{t^2+1}} - \ln \frac{1}{\sqrt{2}}) = \frac{1}{2}\ln 2$$