# 國 立 雲 林 科 技 大 學

109 學年度第一學期工學院會考試題

時間：10:10~12:00
科目：微積分
班級：

※本次測驗禁止使用電子計算機

1. 求函數 $f(x)=x+\frac{1}{x}$ 在區間 [0.1, 5] 上的絕對極大值(absolute maximum)與絕對極小值(absolute minimum)。 (10%)

2. 函數 $f(x)=x(6-x)^{2/3}$，求下列開區間與極值： (30%)

   (a) 遞增(increasing)或遞減(decreasing)的區間。

   (b) 相對極大值(local maximum)與相對極小值(local minimum)。

   (c) 圖形為凹向上(concave up)或凹向下(concave down)的區間。

3. 求以方程式 $y=\left(1+\frac{1}{x}\right)^2$、$x=1$、$x=2$、$y=0$ 的圖形為界之區域面積。 (6%)

4. 求 $F(\theta)=\int_1^{\sin\theta}\sqrt{1+(\arcsin x)^2}\,dx$ 對 $\theta$ 的導數 (derivative)。 (6%)

5. 求下列不定積分： $(6\%\times 5=30\%)$

   (a) $\int\frac{1}{x^2}\left(1+\frac{1}{x}\right)^{1/3}dx$ (b) $\int\frac{x^2-1}{\sqrt{x+2}}dx$ (c) $\int\frac{\ln x}{x\sqrt{1+(\ln x)^2}}dx$

   (d) $\int\frac{xe^{\sqrt{x^2+1}}}{2\sqrt{x^2+1}}dx$ (e) $\int\frac{1}{\sqrt{x}\left(1-3\sqrt{x}\right)}dx$

6. 求下列定積分： $(6\%\times 3=18\%)$

   (a) $\int_0^{1/2}\frac{2x}{\left(1-2x^2\right)^2}dx$ (b) $\int_0^1(1+x)\sqrt{1-x}\,dx$ (c) $\int_0^{\pi/8}\frac{\sec^2 2\theta}{1+\tan 2\theta}d\theta$

《試題完》

# 國 立 雲 林 科 技 大 學

109 學年度第一學期工學院會考試題

時間：10:10~12:00
科目：Calculus
班級：

※ The use of scientific calculators is not permitted in examinations.

1. Find the absolute maximum and absolute minimum values of the function on the given interval. (10%)

$$f(x)=x+\frac{1}{x},\ [0.1,\ 5]$$

2. (a) Find the intervals of increase or decrease. (b) Find the local maximum and minimum values. (c) Find the intervals of concavity. (30%)

$$f(x)=x(6-x)^{2/3}$$

3. Find the area of the region bounded by the graphs of the equations. (6%)

$$y=\left(1+\frac{1}{x}\right)^2,\quad x=1,\quad x=2,\quad y=0$$

4. Find the derivative of $F(\theta)=\int_1^{\sin\theta}\sqrt{1+(\arcsin x)^2}\,dx$ 。 (6%)

5. Find the indefinite integral. (6% × 5 = 30%)

(a) $\int\frac{1}{x^2}\left(1+\frac{1}{x}\right)^{1/3}dx$ (b) $\int\frac{x^2-1}{\sqrt{x+2}}dx$ (c) $\int\frac{\ln x}{x\sqrt{1+(\ln x)^2}}dx$

(d) $\int\frac{xe^{\sqrt{x^2+1}}}{2\sqrt{x^2+1}}dx$ (e) $\int\frac{1}{\sqrt{x}\left(1-3\sqrt{x}\right)}dx$

6. Evaluate the definite integral. (6% × 3 = 18%)

(a) $\int_0^{1/2}\frac{2x}{\left(1-2x^2\right)^2}dx$ (b) $\int_0^1(1+x)\sqrt{1-x}\,dx$ (c) $\int_0^{\pi/8}\frac{\sec^2 2\theta}{1+\tan 2\theta}d\theta$

# 國立雲林科技大學 109 學年度第一學期工學院微積分統一會考

# 標準答案試算表

**No**. 1

1. $f(x) = x + \dfrac{1}{x}, \quad [0.1, 5] \qquad f'(x) = 1 - \dfrac{1}{x^2} = \dfrac{x^2 - 1}{x^2} = \dfrac{(x-1)(x+1)}{x^2},$

The critical numbers: +1, –1, 0 but only +1 is in the given interval [0.1, 5].

$f(0.1) = 10.1; \quad f(1) = 2; \quad f(5) = 5.2$

So $f(0.1) = 10.1$ is the absolute maximum value, and $f(1) = 2$ is the absolute maximum value.

2. $f(x) = x(6-x)^{2/3} \qquad f'(x) = x\dfrac{2}{3}(6-x)^{-1/3}(-1) + (6-x)^{2/3}$

$$= \frac{-\frac{2}{3}x + (6-x)}{(6-x)^{1/3}} = \frac{6 - \frac{5}{3}x}{(6-x)^{1/3}} = \frac{1}{3}\frac{18-5x}{(6-x)^{1/3}}$$

The critical numbers: 18/5 = 3.6 ; and 6

(a)

| Interval | $18 - 5x$ | $(6-x)^{1/3}$ | $f'(x)$ |
|---|---|---|---|
| $x < 3.6$ | + | + | + |
| $3.6 < x < 6$ | – | + | – |
| $x > 6$ | – | – | + |

$f$ is increasing on $(-\infty, 3.6)$ and $(6, \infty)$, and $f$ is decreasing on $(3.6, 6)$.

(b) $f$ changes from increasing to decreasing at $x = 3.6$, so $f(3.6) = 3.6 \times (2.4)^{2/3}$ is a local maximum value.

$f$ changes from decreasing to increasing at $x = 6$, so $f(6) = 0$ is a local minimum value.

(c)

$$f''(x) = \frac{1}{3}\frac{(-5)(6-x)^{1/3} - (18-5x)(1/3)(6-x)^{-2/3}(-1)}{(6-x)^{2/3}} = \frac{1}{3}\frac{(-5)(6-x) + (18-5x)(1/3)}{(6-x)^{4/3}}$$

$$= \frac{1}{3}\frac{-30 + 5x + 6 - 5/3x}{(6-x)^{4/3}} = \frac{1}{3}\frac{-24 + 10/3x}{(6-x)^{4/3}} = \frac{1}{3}\cdot\frac{10}{3}\cdot\frac{(x-7.2)}{(6-x)^{4/3}}$$

$f''(x) > 0$ when $x > 7.2 \Rightarrow$ concave upward on $(7.2, \infty)$

$f''(x) < 0$ when $x < 7.2$ (but $x \neq 6$) $\Rightarrow$ concave downward on $(-\infty, 6)$ and $(6, 7.2)$

# 國立雲林科技大學 109 學年度第一學期工學院微積分統一會考

## 標準答案試算表



3.

Area =

$$\int_1^2\left(1+\frac{1}{x}\right)^2 dx=\int_1^2\left(1+\frac{2}{x}+\frac{1}{x^2}\right)dx=\left[x+2\ln x-\frac{1}{x}\right]_1^2=2+2\ln 2-\frac{1}{2}=\frac{3}{2}+2\ln 2$$

4.

$$\frac{d}{d\theta}\left[\int_1^{\sin\theta}\sqrt{1+(\arcsin x)^2}\,dx\right]=\sqrt{1+[\arcsin(\sin\theta)]^2}\cdot\cos\theta-0=\sqrt{1+\theta^2}\cdot\cos\theta$$

5.

(a) $\int\frac{1}{x^2}\left(1+\frac{1}{x}\right)^{1/3}dx=-\int\left(1+\frac{1}{x}\right)^{1/3}(-x^{-2})dx=-\frac{3}{4}\left(1+\frac{1}{x}\right)^{4/3}+C$

(b) Let $u=x+2$ ， $x=u-2$ ， $dx=du$ ，

$$\int\frac{x^2-1}{\sqrt{x+2}}dx=\int\frac{(u-2)^2-1}{u^{1/2}}du=\int(u^{3/2}-4u^{1/2}+3u^{-1/2})du$$

$$=\frac{2}{5}u^{5/2}-\frac{8}{3}u^{3/2}+6u^{1/2}+C=\frac{2}{5}(x+2)^{5/2}-\frac{8}{3}(x+2)^{3/2}+6(x+2)^{1/2}+C$$

(c) $\int\frac{\ln x}{x\sqrt{1+(\ln x)^2}}dx=\frac{1}{2}\int\left[1+(\ln x)^2\right]^{-1/2}2(\ln x)\frac{1}{x}dx=\sqrt{1+(\ln x)^2}+C$

(d) $\int\frac{xe^{\sqrt{x^2+1}}}{2\sqrt{x^2+1}}dx=\frac{1}{2}\int e^{\sqrt{x^2+1}}\cdot\frac{1}{2}(x^2+1)^{-1/2}(2x)\,dx=\frac{1}{2}e^{\sqrt{x^2+1}}+C$

(e) $\int\frac{1}{\sqrt{x}\left(1-3\sqrt{x}\right)}dx=-\frac{2}{3}\int\frac{1}{\left(1-3\sqrt{x}\right)}\left(-\frac{3}{2\sqrt{x}}\right)dx=-\frac{2}{3}\ln\left|1-3\sqrt{x}\right|+C$

# 國立雲林科技大學 109 學年度第一學期工學院微積分統一會考

# 標準答案試算表

**No.** 3

6.

(a)

$$\int_0^{1/2} \frac{2x}{\left(1-2x^2\right)^2}\,dx = -\frac{1}{2}\int_0^{1/2}\left(1-2x^2\right)^{-2}(-4x)\,dx = -\frac{1}{2}\left[\frac{-1}{1-2x^2}\right]_0^{1/2}$$

$$= \frac{1}{2}\left(\frac{1}{1/2}-\frac{1}{2}\right) = \frac{1}{2}$$

(b) Let $u = 1-x$ ， $x = 1-u$ ， $dx = -du$ ，

$$\int_0^1 (1+x)\sqrt{1-x}\,dx = \int_1^0 (1+1-u)\sqrt{u}\,(-du) = \int_0^1 (2u^{1/2}-u^{3/2})\,du$$

$$= \left[\frac{4}{3}u^{3/2}-\frac{2}{5}u^{5/2}\right]_0^1 = \frac{4}{3}-\frac{2}{5} = \frac{14}{15}$$

(c)

$$\int_0^{\pi/8} \frac{\sec^2 2\theta}{1+\tan 2\theta}\,d\theta = \frac{1}{2}\int_0^{\pi/8} \frac{\sec^2 2\theta}{1+\tan 2\theta}\,2d\theta = \frac{1}{2}\left[\ln\left|1+\tan 2\theta\right|\right]_0^{\pi/8}$$

$$= \frac{1}{2}(\ln 2-\ln 1) = \frac{1}{2}\ln 2$$