國 立 雲 林 科 技 大 學

108 學年度第一學期工學院會考試題

時間：10:10~12:00
科目：微積分
班級：

※本次測驗禁止使用電子計算機與手機

1. 求函數 $f(x)$ 在給定區間內的絕對極大值（absolute maximum values）和絕對極小值（absolute minimum values）（每小題 10 分，此題共 20 分）

(a) $f(x)=\dfrac{3x-4}{x^2+1},\quad -2\le x\le 2$

(b) $f(x)=\cos^2 x-2\sin x,\quad 0\le x\le 2\pi$

2. x-y 平面上之曲線 $x^2+xy+y^2=12$，

(a) 求 $\dfrac{dy}{dx}$ 之表示式。（4 分）

(b) 求曲線上的最高點與最低點座標。（6 分）

3. $y=\dfrac{x^2}{x-1}$，求（每小題 4 分，此題共 20 分）

(a) 此函數圖形的所有漸近線（提示：此函數有斜漸近線）。

(b) 此函數遞增（increasing）與遞減（decreasing）的區間。

(c) 函數所有的相對極大值（local maximum）、相對極小值（local minimum）及反曲點（inflection point）。

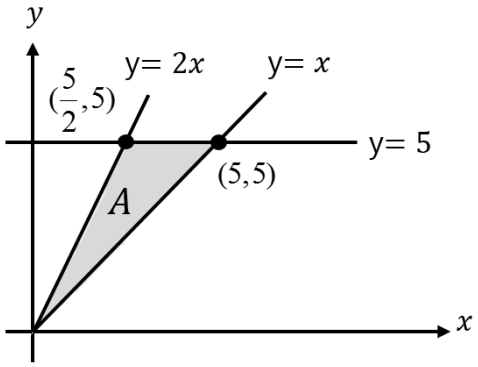
(d) 函數圖形凹向上（concave upward）及凹向下（concave downward）的區間。

(e) 於 x-y 平面上繪製此函數的圖形。

《背面尚有試題》

4. 試計算以方程式$y=x$、$y=2x$、$y=5$的圖形所圍出區域A的面積。請參考下圖。

(請使用積分法求解) (此題共5分）



5. 試計算出$\frac{d}{dx}\left[\int_{x^2}^{1}(2t-\sin^2 t)dt\right]$。 (此題共5分）

6. 試計算下列函數的定積分： （每小題5分，此題共10分）

(a) $\int_1^5 \frac{10x^4+2}{x^5+x+2}dx$ (b) $\int_0^{\sqrt[5]{5\ln 7}} x^4 e^{-\frac{x^5}{5}}dx$

7. 計算下列函數的不定積分： （每小題6分，此題共30分）

(a) $\int \frac{4}{(4x-1)^2}dx$ (b) $\int \frac{1}{\sqrt{x\sqrt{x}+x}}dx$ (c) $\int(1+\tan^2 x)\sec^2 x dx$

(d) $\int \frac{e^{2x}}{2e^{2x}+3}dx$ (e) $\int \sec 2x dx$

《試題完》

國立雲林科技大學 108 學年度第一學期工學院微積分統一會考

標準答案試算表

1.

1(a)

$f(x)=\dfrac{3x-4}{x^2+1},\ [-2,2].\quad f'(x)=\dfrac{(x^2+1)(3)-(3x-4)(2x)}{(x^2+1)^2}=\dfrac{-(3x^2-8x-3)}{(x^2+1)^2}=\dfrac{-(3x+1)(x-3)}{(x^2+1)^2}.$

$f'(x)=0 \ \Rightarrow\ x=-\frac{1}{3}$ or $x=3$, but 3 is not in the interval. $f'(x)>0$ for $-\frac{1}{3}<x<2$ and $f'(x)<0$ for $-2<x<-\frac{1}{3}$, so $f\left(-\frac{1}{3}\right)=\frac{-5}{10/9}=-\frac{9}{2}$ is a local minimum value. Checking the endpoints, we find $f(-2)=-2$ and $f(2)=\frac{2}{5}$. Thus, $f\left(-\frac{1}{3}\right)=-\frac{9}{2}$ is the absolute minimum value and $f(2)=\frac{2}{5}$ is the absolute maximum value.

1(b).

$f(x)=\cos^2 x-2\sin x,\ \ 0\le x\le 2\pi.\quad f'(x)=-2\cos x\sin x-2\cos x=-2\cos x\,(1+\sin x)$. Note that $1+\sin x\ge 0$ [since $\sin x\ge -1$], with equality $\Leftrightarrow\ \sin x=-1\ \Leftrightarrow\ x=\frac{3\pi}{2}$ [since $0\le x\le 2\pi$] $\Rightarrow$ $\cos x=0$. Thus, $f'(x)>0\ \Leftrightarrow\ \cos x<0\ \Leftrightarrow\ \frac{\pi}{2}<x<\frac{3\pi}{2}$ and $f'(x)<0\ \Leftrightarrow\ \cos x>0\ \Leftrightarrow\ 0<x<\frac{\pi}{2}$ or $\frac{3\pi}{2}<x<2\pi$. Thus, f is increasing on $\left(\frac{\pi}{2},\frac{3\pi}{2}\right)$ and f is decreasing on $\left(0,\frac{\pi}{2}\right)$ and $\left(\frac{3\pi}{2},2\pi\right)$.

f changes from decreasing to increasing at $x=\frac{\pi}{2}$ and from increasing to decreasing at $x=\frac{3\pi}{2}$. Thus, $f\left(\frac{\pi}{2}\right)=-2$ is a local minimum value and $f\left(\frac{3\pi}{2}\right)=2$ is a local maximum value.

端點 $f(0)=\cos^2 0-2\sin 0=1;\qquad f(2\pi)=\cos^2(2\pi)-2\sin(2\pi)=1$

絕對極小: $f\left(\dfrac{\pi}{2}\right)=-2$ 絕對極大 $f\left(\dfrac{3\pi}{2}\right)=2$

2.

2(a)(b)

Differentiating $x^2+xy+y^2=12$ implicitly with respect to x gives $2x+y+x\dfrac{dy}{dx}+2y\dfrac{dy}{dx}=0$, so $\dfrac{dy}{dx}=-\dfrac{2x+y}{x+2y}$.

At a highest or lowest point, $\dfrac{dy}{dx}=0\ \Leftrightarrow\ y=-2x$. Substituting $-2x$ for y in the original equation gives $x^2+x(-2x)+(-2x)^2=12$, so $3x^2=12$ and $x=\pm 2$. If $x=2$, then $y=-2x=-4$, and if $x=-2$ then $y=4$. Thus, the highest and lowest points are $(-2,4)$ and $(2,-4)$.

3.

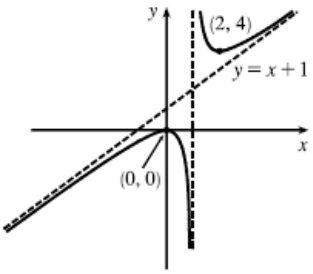
3(a)~(e)

$y = f(x) = \dfrac{x^2}{x-1} = x + 1 + \dfrac{1}{x-1}$ **A.** $D = (-\infty, 1) \cup (1, \infty)$ **B.** x-intercept: $f(x) = 0 \Leftrightarrow x = 0$; y-intercept: $f(0) = 0$ **C.** No symmetry **D.** $\lim\limits_{x \to 1^-} f(x) = -\infty$ and $\lim\limits_{x \to 1^+} f(x) = \infty$, so $x = 1$ is a VA.

$\lim\limits_{x \to \pm\infty} [f(x) - (x+1)] = \lim\limits_{x \to \pm\infty} \dfrac{1}{x-1} = 0$, so the line $y = x + 1$ is a SA.

E. $f'(x) = 1 - \dfrac{1}{(x-1)^2} = \dfrac{(x-1)^2 - 1}{(x-1)^2} = \dfrac{x^2 - 2x}{(x-1)^2} = \dfrac{x(x-2)}{(x-1)^2} > 0$ for $x < 0$ or $x > 2$, so f is increasing on $(-\infty, 0)$ and $(2, \infty)$, and f is decreasing on $(0, 1)$ and $(1, 2)$. **F.** Local maximum value $f(0) = 0$, local minimum value $f(2) = 4$ **G.** $f''(x) = \dfrac{2}{(x-1)^3} > 0$ for $x > 1$, so f is CU on $(1, \infty)$ and f is CD on $(-\infty, 1)$. No IP

H.



4.

$$A = \int_0^{\frac{5}{2}} [2x - x]\,dx + \int_{\frac{5}{2}}^{5} [5 - x]\,dx$$

$$= \left[\frac{1}{2}x^2\right]_0^{\frac{5}{2}} + \left[5x - \frac{1}{2}x^2\right]_{\frac{5}{2}}^{5}$$

$$= \frac{25}{8} + [\frac{25}{2} - \frac{75}{8}] = \frac{50}{8}$$

5.

$F(x) = \int_{x^2}^{1} (2t - \sin^2 t)dt = -\int_1^{x^2} (2t - \sin^2 t)dt$. Let $u = x^2$. Then

$$\frac{dF}{dx} = \frac{dF}{du}\frac{du}{dx}$$

$$= -(2u - \sin^2 u)(2x)$$

$$= -(2x^2 - \sin^2(x^2))(2x)$$

$$= (2x)[\sin^2(x^2) - 2x^2]$$

6.

6(a)

Set $u = x^5 + x + 2$. Then $du = (5x^4 + 1)dx$.

At $x = 1$, $u = 4$. At $x = 5$, $u = 3125$. Thus

$$\int_1^5 \frac{10x^4 + 2}{x^5 + x + 2} dx = \int_1^5 \frac{2(5x^4 + 1)}{x^5 + x + 2} dx = 2\int_1^5 \frac{1}{u} du = 2\left[\ln|u|\right]_4^{3125}$$
$$= 2(\ln 3125 - \ln 4) = 2\ln(\frac{3125}{4})$$

6(b)

Set $u = -\frac{x^5}{5}$. Then $du = -x^4 dx$.

At $x = 0$, $u = 0$. At $x = \sqrt[5]{5\ln 7}$, $u = -\ln 7$. Thus

$$\int_0^{\sqrt[5]{5\ln 7}} x^4 e^{-\frac{x^5}{5}} dx = -\int_0^{-\ln 7} e^u du = -\left[e^u\right]_0^{-\ln 7} = 1 - e^{-\ln 7} = \frac{6}{7}$$

7.

7(a)

$$\int \frac{4}{(4x-1)^2} dx = \int 4(4x-1)^{-2} dx = \int (4x-1)^{-2} d(4x-1) = -(4x-1)^{-1} + C$$

7(b)

Set $u = \sqrt{x} + 1$. Then $du = \frac{1}{2\sqrt{x}} dx$. Thus

$$\int \frac{1}{\sqrt{x\sqrt{x} + x}} dx = \int \frac{1}{\sqrt{x(\sqrt{x} + 1)}} dx = \int \frac{1}{\sqrt{x}\sqrt{\sqrt{x} + 1}} dx = 2\int \frac{1}{u^{\frac{1}{2}}} du = 2\int u^{-(\frac{1}{2})} du$$
$$= 4\sqrt{u} + C = 4\sqrt{\sqrt{x} + 1} + C$$

7(c)

Set $u = \tan x$. Then $du = \sec^2 x dx$. Thus

$$\int (1 + \tan^2 x)\sec^2 x dx = \int \sec^2 x dx + \int \sec^2 x \tan^2 x dx = u + \int u^2 du$$
$$= u + \frac{1}{3}u^3 + C = \tan x + \frac{1}{3}\tan^3 x + C$$

7(d)

Set $u = 2e^{2x} + 3$. Then $du = 4e^{2x}dx$. Thus

$$\int \frac{e^{2x}}{2e^{2x}+3}dx = \frac{1}{4}\int \frac{1}{u}du = \frac{1}{4}\ln u + C = \frac{1}{4}\ln(2e^{2x}+3) + C$$

7(e)

As we can check, $\frac{1}{2}\ln\left|\sec 2x + \tan 2x\right|$ is an antiderivative for $\sec 2x$.

Therefore

$$\int \sec 2x dx = \frac{1}{2}\int \sec 2x d(2x) = \frac{1}{2}\int \sec 2x \frac{\sec 2x + \tan 2x}{\sec 2x + \tan 2x} d(2x) = \frac{1}{2}\int \frac{\sec^2 2x + \sec 2x \tan 2x}{\sec 2x + \tan 2x} d(2x)$$

Set $u = \sec 2x + \tan 2x$. Then $du = (\sec^2 2x + \sec 2x \tan 2x)d(2x)$. Thus

$$\frac{1}{2}\int \frac{\sec^2 2x + \sec 2x \tan 2x}{\sec 2x + \tan 2x} d(2x) = \frac{1}{2}\int \frac{1}{u} d(u) = \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln\left|\sec 2x + \tan 2x\right| + C$$

《試題解答完》