國 立 雲 林 科 技 大 學

107 學年度第一學期工學院會考試題

時間：10:10~12:00
科目：微積分
班級：

※本次測驗禁止使用電子計算機

1. 求下列函數圖形上的特點：(1) 定義域 (domain)；(2) 相對(局部)極值 (relative extrema)；(3) 反曲點 (inflection points)；(4) 水平或鉛直漸近線 (horizontal or vertical asymptotes)。如果沒有該種特點，請寫 “No”。 (12% × 4 = 48%)

   (a) $f(x) = x^4 - 4x^3$

   (b) $f(x) = \dfrac{x}{(x-1)^2}$

   (c) $f(x) = x - 3x^{1/3}$

   (d) $f(x) = x \ln x$

2. 求以方程式 $y = 1 + x^{1/3}$、$x = 0$、$x = 8$、$y = 0$ 的圖形為界之區域面積。 (5%)

3. 求 $F(x) = \int_0^{x^3} \sin t^2 dt$ 對 x 的導數 (derivative)。 (5%)

4. 求下列不定積分： (6% × 5 = 30%)

   (a) $\int \dfrac{7x}{(1-x^2)^3} dx$ (b) $\int x\sqrt{2x-1}\, dx$ (c) $\int e^{2x}(e^{2x}+1)^6 dx$

   (d) $\int \dfrac{1}{x^{2/3}(1+x^{1/3})} dx$ (e) $\int e^{-x} \tan(e^{-x}) dx$

5. 求下列定積分： (6% × 2 = 12%)

   (a) $\int_1^2 2x^2\sqrt{x^3+1}\ dx$ (b) $\int_1^e \dfrac{(1+\ln x)^2}{x} dx$

《試題完》

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標準答案試算表

No. 1

1.

(a) Ans: (1) $(-\infty, \infty)$ 也可以寫 R

(2) global minimum $f(3) = -27$

(3) (0,0)與(2, −16)

(4) No

(b) Ans: (1) $\{x|x \neq 1\}$ 也可以寫 $(-\infty, 1) \cup (1, \infty)$

(2) local minimum $f(-1) = \frac{-1}{4}$

(3) $(-2, \frac{-2}{9})$

(4) horizontal asymptote: $y = 0$; vertical asymptote: $x = 1$

$y = x/(x-1)^2$ **A.** $D = \{x \mid x \neq 1\} = (-\infty, 1) \cup (1, \infty)$ **B.** x-intercept $= 0$, y-intercept $= f(0) = 0$

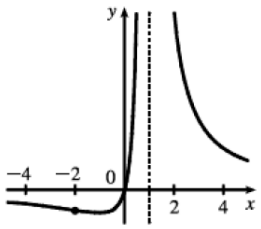
C. No symmetry **D.** $\lim_{x \to \pm\infty} \frac{x}{(x-1)^2} = 0$, so $y = 0$ is a HA. $\lim_{x \to 1} \frac{x}{(x-1)^2} = \infty$, so $x = 1$ is a VA.

E. $f'(x) = \frac{(x-1)^2(1) - x(2)(x-1)}{(x-1)^4} = \frac{-x-1}{(x-1)^3}$. This is negative on $(-\infty, -1)$ and $(1, \infty)$ and positive on $(-1, 1)$, so $f(x)$ is decreasing on $(-\infty, -1)$ and $(1, \infty)$ and increasing on $(-1, 1)$.

F. Local minimum value $f(-1) = -\frac{1}{4}$, no local maximum.

G. $f''(x) = \frac{(x-1)^3(-1) + (x+1)(3)(x-1)^2}{(x-1)^6} = \frac{2(x+2)}{(x-1)^4}$. This is negative on $(-\infty, -2)$, and positive on $(-2, 1)$ and $(1, \infty)$. So f is CD on $(-\infty, -2)$ and CU on $(-2, 1)$ and $(1, \infty)$. IP at $(-2, -\frac{2}{9})$

H.



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標準答案試算表

No. 2

(c) Ans: (1) $(-\infty, \infty)$ 也可以寫 R

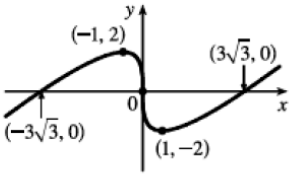
(2) local maximum $f(-1) = 2$ ； local minimum $f(1) = -2$

(3) (0,0)

(4) No

$y = f(x) = x - 3x^{1/3}$ **A.** $D = \mathbb{R}$ **B.** y-intercept: $f(0) = 0$; x-intercepts: $f(x) = 0 \Rightarrow x = 3x^{1/3} \Rightarrow x^3 = 27x \Rightarrow x^3 - 27x = 0 \Rightarrow x(x^2 - 27) = 0 \Rightarrow x = 0, \pm 3\sqrt{3}$ **C.** $f(-x) = -f(x)$, so f is odd; the graph is symmetric about the origin. **D.** No asymptote **E.** $f'(x) = 1 - x^{-2/3} = 1 - \frac{1}{x^{2/3}} = \frac{x^{2/3} - 1}{x^{2/3}}$. $f'(x) > 0$ when $|x| > 1$ and $f'(x) < 0$ when $0 < |x| < 1$, so f is increasing on $(-\infty, -1)$ and $(1, \infty)$, and decreasing on $(-1, 0)$ and $(0, 1)$ [hence decreasing on $(-1, 1)$ since f is continuous on $(-1, 1)$]. **F.** Local maximum value $f(-1) = 2$, local minimum value $f(1) = -2$ **G.** $f''(x) = \frac{2}{3}x^{-5/3} < 0$ when $x < 0$ and $f''(x) > 0$ when $x > 0$, so f is CD on $(-\infty, 0)$ and CU on $(0, \infty)$. IP at $(0, 0)$

H.



(d) Ans: (1) $(0, \infty)$ 也可以寫 $x > 0$

(2) local minimum $f(1/e) = -1/e$

(3) No

(4) No

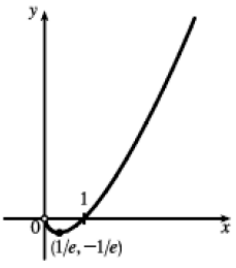
$y = f(x) = x \ln x$ **A.** $D = (0, \infty)$ **B.** x-intercept when $\ln x = 0 \Leftrightarrow x = 1$, no y-intercept **C.** No symmetry

D. $\lim_{x \to \infty} x \ln x = \infty$, $\lim_{x \to 0+} x \ln x = \lim_{x \to 0+} \frac{\ln x}{1/x} \overset{H}{=} \lim_{x \to 0+} \frac{1/x}{-1/x^2} = \lim_{x \to 0+} (-x) = 0$, no asymptote.

E. $f'(x) = \ln x + 1 = 0$ when $\ln x = -1 \Leftrightarrow x = e^{-1}$.

$f'(x) > 0 \Leftrightarrow \ln x > -1 \Leftrightarrow x > e^{-1}$, so f is increasing on $(1/e, \infty)$ and decreasing on $(0, 1/e)$. **F.** $f(1/e) = -1/e$ is an absolute and local minimum value. **G.** $f''(x) = 1/x > 0$, so f is CU on $(0, \infty)$. No IP

H.



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標準答案試算表

No. 3

2.

$$\text{Area} = \int_0^8 (1+x^{1/3})dx = \left[x + \frac{3}{4}x^{4/3} \right]_0^8 = 8 + \frac{3}{4}(8)^{4/3} = 8 + 12 = 20$$

3.

$$\frac{d}{dx}\left[\int_0^{x^3} \sin t^2 dt \right] = \sin(x^3)^2 \times 3x^2 - 0 = 3x^2 \sin x^6$$

4.

(a) $\int \frac{7x}{(1-x^2)^3} dx = \frac{-7}{2}\int (1-x^2)^{-3}(-2x)dx = \frac{-7}{2} \cdot \frac{(1-x^2)^{-2}}{-2} = \frac{7}{4(1-x^2)^2} + C$

(b) 令 $u = 2x-1$ ， $x = \frac{u+1}{2}$ ， $dx = \frac{du}{2}$ ，

$$\int x\,(2x-1)^{1/2} dx = \int \frac{u+1}{2} \cdot u^{1/2} \cdot \frac{du}{2} = \frac{1}{4}\int (u^{3/2} + u^{1/2})du = \frac{1}{4}\left(\frac{2}{5}u^{5/2} + \frac{2}{3}u^{3/2} \right) + C$$

$$= \frac{1}{10}(2x-1)^{5/2} + \frac{1}{6}(2x-1)^{3/2} + C$$

(c) $\int e^{2x}(e^{2x}+1)^6 dx = \frac{1}{2}\int (e^{2x}+1)^6 2e^{2x} dx = \frac{1}{2} \cdot \frac{(e^{2x}+1)^7}{7} = \frac{(e^{2x}+1)^7}{14} + C$

(d) $\int \frac{1}{x^{2/3}(1+x^{1/3})} dx = 3\int \frac{1}{(1+x^{1/3})} \frac{1}{3} x^{-2/3} dx = 3\ln\left|1+x^{1/3}\right| + C$

(e) $\int e^{-x} \tan(e^{-x}) dx = \int \frac{-\sin(e^{-x})}{\cos(e^{-x})}(-e^{-x})\,dx = \ln\left|\cos(e^{-x})\right| + C$

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標準答案試算表

5.

(a)

$$\int_1^2 2x^2\sqrt{x^3+1}\,dx = \frac{2}{3}\int_1^2 (x^3+1)^{1/2}(3x^2)\,dx = \frac{2}{3}\left[\frac{(x^3+1)^{3/2}}{3/2}\right]_1^2$$

$$= \frac{4}{9}(9^{3/2}-2^{3/2}) = 12-\frac{8}{9}\sqrt{2}$$

(b)

$$\int_1^e \frac{(1+\ln x)^2}{x}\,dx = \int_1^e (1+\ln x)^2\frac{1}{x}\,dx = \frac{(1+\ln x)^3}{3}\Big]_1^e = \frac{2^3}{3}-\frac{1}{3} = \frac{7}{3}$$