	國 立 雲 林 科 技 大 學	時間：10:10~12:00
		科目：微積分
	113 學年度第一學期工學院會考試題	班級：

※本次會考禁止使用電子計算機

1. 函數 $f(x)=2\cos^2(x)+\sin^2(2x)$ ，求函數在 $[0,\frac{\pi}{2}]$ 區間上的絕對最大值與絕對最小值。(10%)

2. 求 $\lim_{x\to 0^+}(4x+1)^{\cot x}$ 。(10%)

3. 找出函數 $f(x)=\dfrac{1}{x(x-5)^2}$ 的(a)漸近線、(b)遞增與遞減之區間、(c)相對極大值與相對極小值、(d)凹向上與凹向下之區間、(e)反曲點，(f)並且畫出函數的圖形。(30%)

4. 求 $\dfrac{d}{dx}\int_2^{\frac{1}{x}}\cos^4 t\,dt$ 。(5%)

5. 求下列積分式：(5% x 9 = 45%)

(a) $\int_0^1 (1+x^2)^3\,dx$

(b) $\int_0^3 (2+\sqrt{9-x^2})\,dx$

(c) $\int_0^4 \left|x^2-9\right|dx$

(d) $\int_0^{\frac{\pi}{4}} \dfrac{(1+\sin\theta)}{\cos^2\theta}\,d\theta$

(e) $\int (x^3+1)(x^4+4x)^4\,dx$

(f) $\int \dfrac{1}{(1+\sqrt{x})^4}\,dx$

(g) $\int \sin(\frac{3}{5}\theta)\,d\theta$

(h) $\int \dfrac{\sin(2x)}{(1+\cos^2 x)}\,dx$

(i) $\int e^{\sec x}\sec x\tan x\,dx$

<<試題完>>

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※ The use of scientific calculators is not permitted in examinations.

1. Find the absolute maximum and absolute minimum values of the function on the given internal. (10%)

$$f(x) = 2\cos^2(x) + \sin^2(2x) \quad [0, \frac{\pi}{2}]$$

2. Evaluate $\lim_{x\to 0^+} (4x+1)^{\cot x}$. (10%)

3. For the function $f(x) = \dfrac{1}{x(x-5)^2}$, find (a) the asymptotes, (b) the intervals of increase or decrease, (c) the local maximum and minimum values, (d) the intervals of concavity, (e) the inflection points, and (f) draw the function on the x-y plane. (30%)

4. Evaluate $\dfrac{d}{dx}\int_2^{\frac{1}{x}} \cos^4 t\,dt$. (5%)

5. Find the indefinite integral. (5% x 9 = 45%)

(a) $\int_0^1 (1+x^2)^3\,dx$ (b) $\int_0^3 (2+\sqrt{9-x^2})\,dx$

(c) $\int_0^4 \left|x^2-9\right| dx$ (d) $\int_0^{\frac{\pi}{4}} \dfrac{(1+\sin\theta)}{\cos^2\theta}\,d\theta$

(e) $\int (x^3+1)(x^4+4x)^4\,dx$ (f) $\int \dfrac{1}{(1+\sqrt{x})^4}\,dx$

(g) $\int \sin(\frac{3}{5}\theta)\,d\theta$ (h) $\int \dfrac{\sin(2x)}{(1+\cos^2 x)}\,dx$

(i) $\int e^{\sec x}\sec x\tan x\,dx$

1 $f(x) = 2\cos^2(x) + \sin^2(2x) \quad , [0, \frac{\pi}{2}]$

$f'(x) = 4\cos x\,(-\sin x) + 2\sin(2x)\cos(2x)\cdot 2$

$= -4\sin x\cos x + 4\sin(2x)\cos(2x)$

$= -4\sin x\cos x + 8\sin x\cos x\cos 2x$

$= 4\sin x\cos x\,[-1 + 2\cos 2x] = 0$

$f'(x) = 0, \quad \sin x = 0 \rightarrow x = 0$

$\cos x = 0 \rightarrow x = \frac{\pi}{2}$

$\cos 2x = \frac{1}{2} \rightarrow 2x = \frac{\pi}{3}, \; x = \frac{\pi}{6}$

$f(0) = 2$

$f(\frac{\pi}{2}) = 0 \rightarrow$ absolute min. $(\frac{\pi}{2}, 0)$

$f(\frac{\pi}{6}) = 2\cdot(\frac{\sqrt{3}}{2})^2 + (\frac{\sqrt{3}}{2})^2 = \frac{9}{4} = 2.25 \rightarrow$ absolute max. $(\frac{\pi}{6}, \frac{9}{4})$

2 $y = (4x+1)^{\cot x} \Rightarrow \ln y = (\cot x)\ln(4x+1)$

$\Rightarrow \lim\limits_{x\to 0^+} \ln y = \lim\limits_{x\to 0^+}\left[(\cot x)\ln(4x+1)\right] = \lim\limits_{x\to 0^+} \frac{\ln(4x+1)}{\tan x} \quad \left(\frac{0}{0}\right)$

$\overset{L'P}{=} \lim\limits_{x\to 0^+} \frac{\frac{4}{4x+1}}{\sec^2 x} = \frac{\frac{4}{1}}{1} = 4$

$\Rightarrow \lim\limits_{x\to 0^+} y = \lim\limits_{x\to 0^+} e^{\ln y} = e^4$

3 $f(x)=\dfrac{1}{x(x-5)^2}$

(a) 水平漸近線 $\lim\limits_{x\to\pm\infty}\dfrac{1}{x(x-5)^2}=0 \to y=0$.

垂直漸近線 $x=0$, $x=5$

$$\lim_{x\to 0^+}\frac{1}{x(x-5)^2}=\infty,\quad \lim_{x\to 0^-}\frac{1}{x(x-5)^2}=-\infty$$

$$\lim_{x\to 5^+}\frac{1}{x(x-5)^2}=\infty,\quad \lim_{x\to 5^-}\frac{1}{x(x-5)^2}=\infty$$

(b)

(c)

$$f'=-x^{-2}(x-5)^{-2}+x^{-1}(-2)(x-5)^{-3}=\frac{-3x+5}{x^2(x-5)^3}$$

關鍵點 $x=0,5,\frac{5}{3}$

	$-\infty$ ~ 0	0 ~ $\frac{5}{3}$	$\frac{5}{3}$ ~ 5	5 ~ ∞
f				
f'	$-$ ↘	$-$ ↘	$+$ ↗	$-$ ↘
		min.		

遞增 $(\frac{5}{3},5)$，遞減 $(-\infty,0)$ $(0,\frac{5}{3})$ $(5,\infty)$

$(\frac{5}{3},\frac{27}{500})$ 最小值

(d)

(e)

$$f''=\frac{-3x^2(x-5)^3-(-3x+5)\left[2x(x-5)^3+x^2\cdot 3(x-5)^2\right]}{x^4(x-5)^6}$$

$$=\frac{-3x(x-5)-\left[(-3x+5)(5x-10)\right]}{x^3(x-5)^4}=\frac{12x^2-40x+50}{x^3(x-5)^4}$$

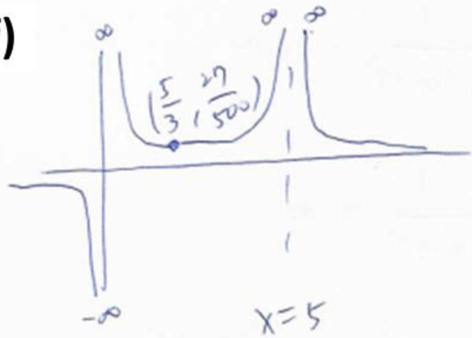
$f''(x)=0 \to x=0,5$

	$-\infty$	0	5	∞
f				
f''	$-\ \cap$	$+\ \cup$	$+\ \cup$	

凹向上 $(0,5)$ $(5,\infty)$　凹向下 $(-\infty,0)$

因 $x=0$ 不在定義域內，無反曲點

(f)



4

$$\frac{d}{dx}\int_2^{1/x}\cos^4 t\,dt = \frac{d}{du}\int_2^{u}\cos^4 t\,dt\left(\frac{du}{dx}\right)$$

$$= \cos^4 u\left(\frac{du}{dx}\right)$$

$$= \frac{-\cos^4(1/x)}{x^2}$$

5(a) $\int_0^1 (1+x^2)^3 dx = \int_0^1 (1+3x^2+3x^4+x^6) dx$

$= \left[x + x^3 + \frac{3}{5}x^5 + \frac{x^7}{7}\right]\Big|_0^1$

$= \frac{96}{35}$

(b) $\int_0^3 (2+\sqrt{9-x^2}) dx = 2\times 3 + \frac{1}{4}\pi \times 3^2$

$= 6 + \frac{9}{4}\pi$

(c) $\int_0^4 |x^2-9| dx = \int_0^3 (9-x^2) dx + \int_3^4 (x^2-9) dx$

$= \left[9x - \frac{x^3}{3}\right]\Big|_0^3 + \left[\frac{x^3}{3} - 9x\right]\Big|_3^4$

$= \frac{64}{3}$

(d) $\int_0^{\pi/4} \frac{(1+\sin\theta)}{\cos^2\theta} d\theta = \int_0^{\pi/4} (\sec^2\theta + \sec\theta\tan\theta) d\theta$

$= (\tan\theta + \sec\theta)\Big|_0^{\pi/4}$

$= \sqrt{2}$

(e) $\int (x^3+1)(x^4+4x)^4 dx = \int u^4 \cdot \left(\frac{du}{4}\right) = \frac{1}{20}u^5 + C$

$= \frac{1}{20}(x^4+4x)^5 + C$

註：令 $u = x^4+4x$

$du = (4x^3+4)dx$

$= 4(x^3+1)dx$

(f) $\int \frac{1}{(1+\sqrt{x})^4}dx = \int \frac{1}{u^4}[2(u-1)du] = 2\int [u^{-3} - u^{-4}]du$

$= -\frac{1}{u^2} + \frac{2}{3u^3} + C$

$= -\frac{1}{(1+\sqrt{x})^2} + \frac{2}{3(1+\sqrt{x})^3} + C$

註：令 $u = 1+\sqrt{x}$

$du = \frac{1}{2\sqrt{x}}dx$

$2(u-1)du = dx$

(g) $\int \sin\left(\frac{3}{5}\theta\right)d\theta = \int \sin u \left(\frac{5}{3}\right)du = -\frac{5}{3}\cos u + C$

$= -\frac{5}{3}\cos\left(\frac{3}{5}\theta\right) + C$

註：令 $u = \frac{3}{5}\theta$

$du = \frac{3}{5}d\theta$

(h) $\int \frac{\sin 2x}{1+\cos^2 x}dx = \int \frac{2\sin x\cos x}{1+\cos^2 x}dx = -\int \frac{2u}{1+u^2}du$

$= \int \frac{(-1)}{t}dt = \ln \frac{1}{1+\cos^2 x}$

註：令 $u = \cos x$, $du = -\sin x$

令 $t = 1+u^2$, $dt = 2udu$

(i) $\int e^{\sec x}\sec x\tan x\,dx = \int e^u du = e^u + C$

$= e^{\sec x} + C$

註：令 $u = \sec x$

$du = \sec x\tan x\,dx$